S&P Dow Jones Indices

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Research

Glass-Box Optimization: Bringing Clarity to Sustainability Indices

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Abstract

This paper investigates the efficacy of S&P Dow Jones Indices' (S&P DJI's) glass-box optimization algorithm for incorporating multiple sustainability-related objectives in the construction of an index. The glass-box optimization underpins the S&P Paris-Aligned & Climate Transition, Sustainability Enhanced, ESG Enhanced and Net Zero Carbon Budget Indices. The approach is motivated by the growing demand for transparency in how sustainability, environmental, social and governance (ESG) and other climate-related objectives are incorporated into the index construction process, and an acknowledgement of the multi-faceted nature of these objectives. The glass-box optimization is compared to a representative risk model-based index optimization in three different scenarios, where each scenario is characterized by a different combination of constraints. Special emphasis is given to the interpretability and explainability of the optimized index weights, with the motivation to minimize the possibility of greenwashing that may be caused by insufficient association between the optimized index weights and the company characteristics used to define the constraints. The results provide strong support in favor of the glass-box optimization as a method for building sustainability indices. The index weights produced by the glass-box optimization are shown to be completely explainable in terms of the constraints, whereas the weights produced by the risk model optimization were strongly influenced by additional factors that are included in the model to explain the covariance matrix of returns. The indices derived by the glass-box optimization also relied less heavily on extreme positions in small, illiquid assets, while achieving similar levels of performance with respect to realized tracking error and lower portfolio turnover.

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1. Introduction

The past decade has been marked by a significant increase in the proportion of wealth held in passive, index-based investment strategies, with 42.9% of assets held by U.S. mutual funds and ETFs now being managed passively, reflecting a 2.3% annualized increase since 2013.[1] At the same time, the demand for sustainability and net zero emissions-aligned investment solutions has increased substantially. The growth in demand for ESG and climate-oriented investment strategies has been driven by the search for long-term financial value and the pursuit of investment opportunities that align with global sustainability objectives.[2] This trend shows no sign of slowing, with PricewaterhouseCoopers (2022) reporting that 8 in 10 investors plan to boost their exposure to ESG over the next two years, and that by 2026, nearly USD 34 trillion in global assets will be directed into ESG funds and other sustainability investment vehicles.[3]

The sustainability characteristics of an index can be improved by applying a series of stock-level selection criteria targeted at removing the least sustainable companies (e.g., business activity exclusions and best-in-class constituent selection) or by defining an index weighting scheme that allocates the greatest weight to companies with the most favorable characteristics (e.g., tilting strategies and constrained index optimization techniques). These steps can be applied individually or together as part of a multi-step methodology. This paper is principally concerned with the latter. Specifically, we consider the problem of improving the sustainability characteristics of a global stock market index by optimizing the weights subject to one or more constraints. This approach is consistent with recent research by Kölbel et al. (2020),[4] who identified capital allocation, shareholder engagement and indirect impacts as the three channels through which sustainability investing contributes to societal goals. The effectiveness of the capital allocation mechanism relies on a strong link between the reweighting of individual companies and the sustainability data used to construct the index. Hence, this paper gives special attention to the strength of the relationship between the index weights and the variables used to define the constraints. Sustainability indices may also serve an important role in their capacity as investment benchmarks—an indirect impact mechanism noted by Kölbel et al. (2020).

1 Seyffart, James, "Passive likely overtakes active by 2026, earlier if bear market," Bloomberg Intelligence, March 11, 2021.

2 OECD, "ESG Investing and Climate Transition: Market Practices, Issues and Policy Considerations," OECD Paris, 2021.

3 PricewaterhouseCoopers (PwC), "Asset and wealth management revolution 2022: Exponential expectations for ESG," 2022.

4 Kölbel, Julian F., Heeb, Florian, Paetzold, Falko and Busch, Timo, "Can Sustainable Investing Save the World? Reviewing the Mechanisms of Investor Impact," Organization & Environment, 33(4), 554–574, 2020.

Constrained optimization has become an increasingly popular alternative to more traditional rules-based approaches for building sustainability indices due to the multi-faceted nature of sustainability, which often necessitates the inclusion of multiple constraints. These methods require the specification of an objective function, which tells the optimization the quantity to maximize or minimize, subject to the constraints. The choice of objective function can significantly affect the composition of the optimized index. Risk model optimizations are a particularly common type of optimization algorithm used for constructing constrained indices. These optimizations typically minimize tracking error by matching the exposure of the constrained and unconstrained indices to various sources of systematic risk. This requires imposing some structure on the covariance matrix of returns, and naturally leads to a solution in which the optimized weights are a function of the identified risk factors. This results in a trade-off between the descriptive validity of the risk model and the transparency with which the constraints dictate the optimized weights: as the model becomes more complex, the relationship between the weights and the constraints becomes less clear. As such, risk model optimizations are akin to black boxes: the weights produced by these models can be a complex function of tens or even hundreds of factors—e.g., quality, value and momentum—which can lead to a situation where the optimized weights are unexplainable in terms of the constraints. This is particularly concerning in the context of sustainability investing because risk model optimizations may increase (decrease) the weight of some low (high) scoring companies to achieve a target risk profile, thereby failing to induce a consistent shift toward more sustainable firms, and potentially misleading investors by masking these positions with a positive headline statistic. Risk model-optimized indices may also fail to adequately incentivize change among the least sustainable firms if the relationship between the weights and the variables that define the constraints is significantly affected by including the additional risk factors in the optimization problem.

In response to these concerns, we propose an alternative index optimization that focuses on minimizing active share subject to the condition of proportional redistribution. This removes the complexity involved in the estimation of a risk model and leads to a set of index weights that are completely explainable in terms of the constraints. Hence, we refer to this model as a “glass box”. We compare the indices produced by the glass-box optimization to those derived using a representative risk model optimization in three different scenarios, where each scenario is characterized by a different combination of two ESG and carbon intensity constraints. In each scenario, we investigate the strength of the relationship between the optimized index weights and the sustainability data used to define the constraints. We also examine the diversification and active share characteristics of the optimized indices, as well as their time-series performance with respect to turnover and realized tracking error.

The remainder of this paper is organized as follows. Section 2 examines the characteristics of an index produced by a risk model optimization subject to a single constraint on ESG score. Section 3 introduces the simple glass-box optimization and compares the index produced by this methodology to the risk model-optimized index derived in Section 2. The analysis is repeated with a single constraint on carbon intensity in Section 4. Section 5 investigates the properties of the optimized indices when both constraints are included in the optimization problem, and Section 6 compares the tracking errors of the optimized indices for each combination of the constraints. Section 7 presents S&P DJI's glass-box optimization, which uses a modified version of the simple glass-box objective function, and compares the indices produced by this methodology with those derived using the simple glass-box and risk model optimizations in the previous sections. Finally, Section 8 concludes.

2. Risk Model Optimization

We first consider the problem of improving the ESG score of an equity benchmark by optimizing the index weights subject to a single weighted-average ESG constraint. The S&P Global LargeMidCap was selected as the benchmark index. The ESG constraint uses S&P DJI ESG Scores and requires a minimum 10% improvement in the weighted-average ESG score of the index. We collected constituent-level data between November 2016 and May 2022. The index was optimized using a representative risk model index optimizer[5] and rebalanced semiannually using data as of the close of the last business day in May and November of each year. The optimized index also incorporated a zero lower bound on the weight of each constituent. Exhibits 1 and 2 report summary statistics for the benchmark and risk model-optimized indices, respectively, as of the last rebalance date of each sample year.

Exhibit 1: Summary Statistics for the S&P Global LargeMidCap

Year[6]	Stock Count	Weight of Top 10	Effective Number of Shares	Weighted-Average ESG Score	Weighted-Average Carbon Intensity
2016	2,951	0.091	436.866	66.688	100.363
2017	2,926	0.099	404.431	63.286	85.826
2018	2,935	0.110	356.082	61.525	88.816
2019	3,309	0.118	317.881	63.175	80.325
2020	3,395	0.148	225.036	52.342	66.804
2021	3,476	0.173	185.037	58.574	56.342
2022	3,476	0.154	206.949	64.672	57.646

Source: S&P Dow Jones Indices LLC. Data as of May 2022 rebalance. Past performance is no guarantee of future results. Table is provided for illustrative purposes.

[5] We use an industry-standard world-wide equity fundamental factor risk model with a medium time horizon for the analysis.

[6] The calculations are based on the November rebalance of the corresponding year, except for 2022 where the calculations are based on the May rebalance. Similar assumptions are applied in all the tables in this paper containing rebalance calculation and labelled using a Year column.

Exhibit 2 reports that the total number of constituents included in the risk model-optimized index ranged between 2,141 and 2,490, representing approximately 73% of the S&P Global LargeMidCap by stock count. The risk model-optimized index also reported a similar effective number of shares to the benchmark, indicating an approximately equal level of index concentration. The sum of the weights of the top 10 constituents by index weight was also similar for the constrained and unconstrained indices. However, despite achieving the target ESG score, the correlation between the proportional changes in weights (proportional redistributions) and ESG scores was markedly low, ranging between just 0.171 and 0.302. The low level of correlation may, in part, be a consequence of the risk model objective function not placing any preference on how index weight is redistributed across constituents. Therefore, we also calculated the correlation between the absolute changes in the index weights and ESG scores for the risk model-optimized index as of each rebalance date. The resulting correlations were indeed higher for all years in the sample. However, the correlations were still relatively low, and the extent of the improvement varies depending on the year considered. For example, correlation increased from 0.252 to 0.644 for 2016, but from 0.188 to just 0.227 for 2021.

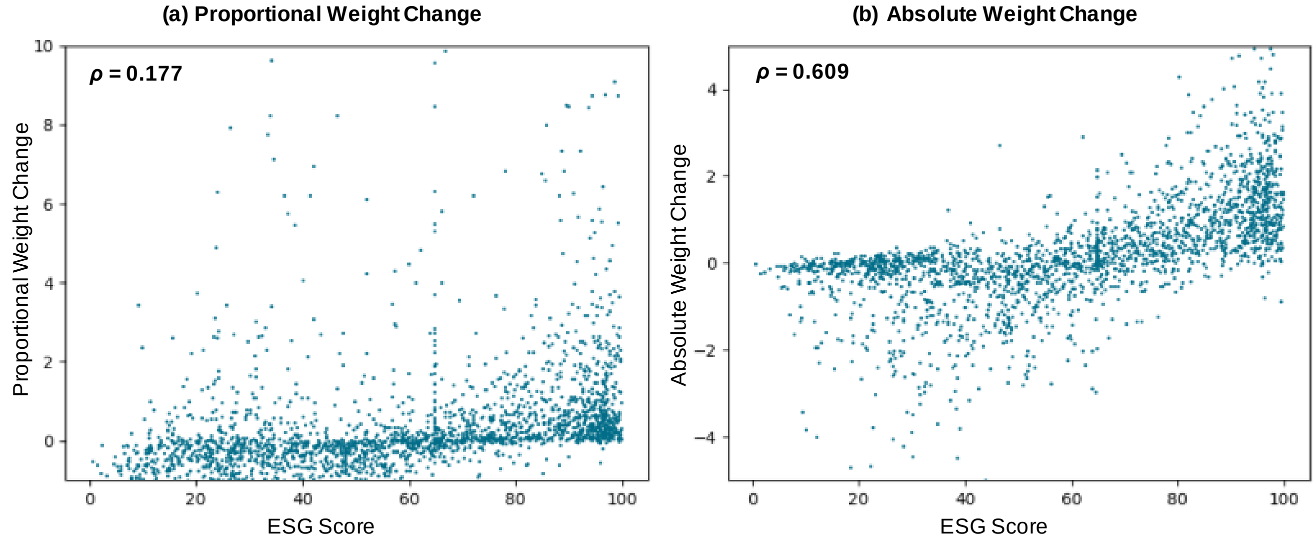
Exhibit 2: Summary Statistics for the Risk Model-Optimized Index with ESG Constraint

Year	Active Share	Stock Count	Weight of Top 10	Effective Number of Shares	Weighted-Average ESG Score	Correlation Weight ESG
2016	0.123	2,141	0.090	423.127	73.357	0.252
2017	0.116	2,185	0.099	393.809	69.595	0.210
2018	0.111	2,218	0.109	350.793	67.625	0.302
2019	0.116	2,307	0.119	309.369	69.493	0.171
2020	0.095	2,490	0.155	213.762	57.573	0.178
2021	0.114	2,408	0.176	177.362	64.426	0.188
2022	0.124	2,174	0.153	204.557	71.129	0.177

The risk model-optimized index is hypothetical.
Source: S&P Dow Jones Indices LLC. Data as of May 2022 rebalance. Correlation Weight ESG is the correlation between the proportional index weight changes and S&P DJI ESG Scores for positively weighted assets. Past performance is no guarantee of future results. Table is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

Exhibit 3 plots the proportional (a) and absolute (b) changes in the weights of the optimized index against the ESG score of each constituent as of the last rebalance date. The plots convey a high degree of dispersion, consistent with the low level of correlation reported in Exhibit 2. The plots also reveal several instances of large positive (negative) changes in weight being applied to low (high) ESG-scoring assets. Therefore, despite an overall increase in the weighted-average ESG score of the index, the weights produced by the risk model optimization algorithm convey a relatively weak relationship with individual ESG scores.

Exhibit 3: Relationship between Proportional and Absolute Weight Changes and ESG Scores – Risk Model-Optimized Index



The risk model-optimized index is hypothetical.
Source: S&P Dow Jones Indices LLC. Data as of May 2022 rebalance. The proportional weight change is defined as the constrained weight minus the unconstrained weight, divided by the unconstrained weight. The range of values displayed in each plot is limited to aid visual comparisons. Past performance is no guarantee of future results. Charts are provided for illustrative purposes and reflect hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

The results conveyed in Exhibits 2 and 3 suggest that the ESG constraint is not the only factor affecting the index weights produced by the risk model optimization. Indeed, the risk model optimization seeks to minimize expected tracking error by matching the risk exposure of the constrained and unconstrained indices with respect to a risk model for returns. The risk model takes the form of a multi-factor model, in which factors for size, quality, value, momentum and volatility are added to a global market risk factor to explain the cross-sectional and time-series properties of asset returns. By trying to match the risk exposure of the constrained and unconstrained indices, the risk model optimizer redistributes the weight across constituents according to each asset's individual risk characteristics, thereby weakening the association between the optimized weights and the variables used to define the constraints; in this case the ESG scores. The interference induced by these additional factors negatively affects the interpretability and explainability of the risk model-optimized index weights, resulting in multiple weights that are unjustifiable with respect to the original constraints. This is especially problematic for sustainability indices, in which stock-level exposure to specific companies is more closely scrutinized than in other more traditionally constructed indices. A strong link between the final weights and the constraints is also necessary for the capital allocation mechanism identified by Kölbel et al. (2020) to effectively incentivize change among the least sustainable firms: a weak relationship between the weights and the data used to define the constraints prevents the efficient reallocation of capital away from unsustainable companies. This motivates an alternative approach where the data used to define the constraints has a more direct influence on the weights of the optimized index.

3. Simple Glass-Box Optimization

The simple glass-box optimization minimizes the sum of the squared differences between the constrained and unconstrained index weights, divided by the unconstrained index weights, such that

$$w^* \equiv \underset{\mathbf{w}^* \in S}{\operatorname{argmin}} \sum_{i=1}^{N} \left(\frac{(w_i^* - w_i)^2}{w_i} \right) \tag{1}$$

where, w_i^* and w_i are the constrained and unconstrained index weights of asset i, respectively, and S is the feasible set defined by the constraints. The set of constraints can include constituent, group and index-level constraints. Division by the unconstrained weights in the objective function ensures proportional redistribution of weight across constituents. The motivation to include this condition is two-fold: managing liquidity and uniformly incentivizing change by proportionally redistributing capital across constituents. The simple glass-box approach can also be embedded into more general optimization problems that include minimum weight thresholds, maximum turnover limits and target stock counts.

Unlike the risk model methodology, the simple glass-box optimization does not require the specification of an explicit model for the relationship between risk and return. Instead, the optimization attempts to minimize active share, subject to the condition of proportional redistribution. The advantage of this approach is that the optimized weights are not a function of the estimated variance-covariance matrix of returns. Instead, the weights are dictated solely by the constraints. One potential limitation of this approach is that it may omit several factors that are important for explaining returns, such that we should not expect an index derived using this methodology to achieve the same tracking error as one produced by a risk model. However, for highly diversified indices, active share and tracking error are closely related quantities, such that minimizing one is likely to come close to minimizing the other. The tracking error of the simple glass-box and risk model-optimized indices is examined in Section 6. The glass-box objective also provides the additional advantage of not requiring estimates of any model parameters, which is of greatest benefit when parameter estimates are least reliable (e.g., during periods of high volatility). Exhibit 4 reports summary statistics for the simple glass-box index subject to the same weighted-average ESG constraint as that described in Section 2.

Exhibit 4: Summary Statistics for the Simple Glass-Box Index with ESG Constraint

Year	Active Share	Stock Count	Weight of Top 10	Effective Number of Shares	Weighted-Average ESG Score	Correlation Weight ESG
2016	0.108	2,951	0.096	436.866	73.356	1.000
2017	0.101	2,926	0.099	404.431	69.614	1.000
2018	0.097	2,935	0.107	356.082	67.678	1.000
2019	0.101	3,309	0.117	317.881	69.493	1.000
2020	0.075	3,395	0.151	225.036	57.576	1.000
2021	0.089	3,476	0.176	185.037	64.431	1.000
2022	0.105	3,476	0.156	206.949	71.130	1.000

The simple glass-box index is hypothetical.
Source: S&P Dow Jones Indices LLC. Data as of May 2022 rebalance. Correlation Weight ESG is the correlation between the proportional changes in index weights and S&P DJI ESG Scores for positively weighted assets. Past performance is no guarantee of future results. Table is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

The statistics conveyed in Exhibit 4 show that the simple glass-box index achieved the target ESG score improvement with comparatively little impact on portfolio composition, as reflected by a lower active share for all sample dates. This is partly attributable to the fact that the index includes all assets in the benchmark universe, thereby offering a more diversified exposure than the risk model-optimized index.

However, the most striking difference between the two approaches is the level of correlation between the proportional redistributions and ESG scores: the simple glass-box approach demonstrates perfect positive correlation compared to the low level of correlation reported for the risk model-optimized index in Exhibit 2. This implies that the simple glass-box methodology is equivalent to a linear tilting scheme with a single weighted-average constraint. To see why, consider the simple glass-box optimization problem with a single constraint on ESG score:

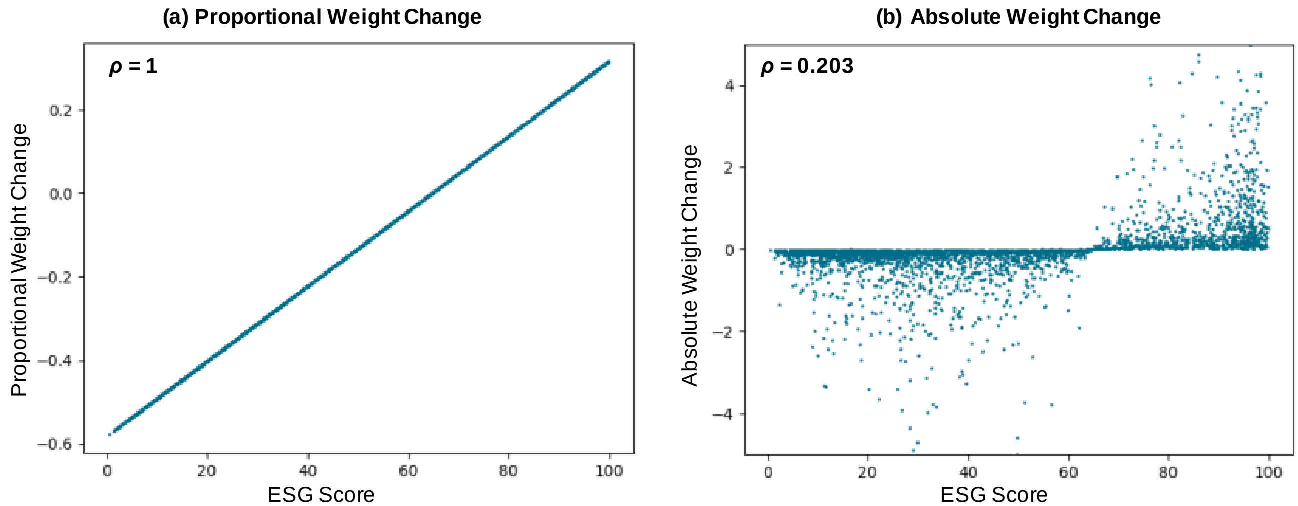
$$\min_{\mathbf{w}^* \geq \mathbf{0}} \sum_{i=1}^{N} \left(\frac{(w_i^* - w_i)^2}{w_i} \right) \quad \text{s.t.} \quad \sum_{i=1}^{N} w_i^* \mathrm{ESG}_i \geq m \quad \text{and} \quad \sum_{i=1}^{N} w_i^* = 1$$

where ESG_i is the ESG score of asset i, and m is the target weighted-average ESG score of the index. If $w_i^* > 0$ for all assets, then the optimized proportional redistributions are given by

$$\frac{w_i^* - w_i}{w_i} = \lambda \left(\mathrm{ESG}_i - \sum_{i=1}^{N} w_i \mathrm{ESG}_i \right) \qquad (2)$$

If the target ESG score is greater than (less than) the benchmark's weighted-average ESG score, then λ will be greater than (less than) zero. A complete description of λ is available in Appendix II. If $\lambda > 0$, then the proportional redistribution applied to company i will be greater than (less than) zero if its ESG score is greater than (less than) the weighted-average ESG score of the benchmark. The extent to which the weights are tilted away from their unconstrained values depends on the magnitude of the required improvement and the variance of ESG scores. Despite their apparent equivalence, there are two main differences between the glass-box optimization and a simple tilting scheme. First, tilting strategies specify the tilting factor λ, whereas the optimization solves for λ as a function of the target value, m. Second, unlike a simple tilting scheme, the glass-box methodology easily generalizes to include multiple constraints.

Exhibit 5: Relationship between Proportional and Absolute Weight Changes and ESG Scores – Simple Glass-Box Index



The simple glass-box index is hypothetical.
Source: S&P Dow Jones Indices LLC. Data as of May 2022 rebalance. The proportional weight change is defined as the constrained weight minus the unconstrained weight, divided by the unconstrained weight. Past performance is no guarantee of future results. Charts are provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

Exhibit 5 plots the proportional (a) and absolute (b) redistributions of the simple glass-box index against the ESG scores for the last rebalance date. Consistent with Equation 2, assets with ESG scores greater than (less than) the weighted-average ESG score of the benchmark (64.67) experienced an increase (decrease) in weight relative to their weight in the benchmark, and the relationship between the proportional redistributions and ESG scores is perfectly linear. The reason we do not observe the same linear relationship in the absolute changes in weights is because they depend on the ESG scores and the unconstrained weights. For example, if two assets have the same ESG score but different benchmark weights, then the asset with the greater benchmark weight will experience a greater absolute change as part of the optimization. This is a consequence of the condition of proportional redistribution embedded in the glass-box objective function. However, the absolute weight change of each asset is still completely explainable in terms of its unconstrained weight and its ESG score.

To see this, multiply both sides of Equation 2 by the benchmark weight to get

$$w_i^* - w_i = \lambda\left(\mathrm{ESG}_i - \sum_{i=1}^{N} w_i \mathrm{ESG}_i\right) w_i \tag{3}$$

Hence, the difference between the optimized and benchmark weight of asset i is proportional to its ESG score and its benchmark weight. Next, we compare the number of observations in each quadrant of the plots in Exhibits 3 and 5, where the quadrants are defined by dividing the vertical axis into values above and below zero and by dividing ESG scores into values above and below the target ESG score. The results are conveyed in Exhibit 6.

Exhibit 6: Quadrant Stock Counts for the Simple Glass-Box and Risk Model-Optimized Indices with ESG Constraints

Simple Glass-Box Index	ESG Score < Target ESG Score	ESG Score > Target ESG Score
Optimized Weight > Benchmark Weight	148	874
Benchmark Weight > Optimized Weight	2,454	0
Risk Model-Optimized Index	**ESG Score < Target ESG Score**	**ESG Score > Target ESG Score**
Optimized Weight > Benchmark Weight	463	813
Benchmark Weight > Optimized Weight	2,138	62

All indices are hypothetical.
Source: S&P Dow Jones Indices LLC. Data as of May 2022 rebalance. Past performance is no guarantee of future results. Table is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

We can quantify the strength of association between the optimized redistributions and ESG scores using the quadrant count ratio (QCR), which is defined as

$$\mathrm{QCR} = \frac{N_1 + N_3 - N_2 - N_4}{N}$$

Where N_1, N_2, N_3, N_4 are the number of observations in the top right, top left, bottom left and bottom right quadrants, respectively, and $N = N_1 + N_2 + N_3 + N_4$. The QCR is robust to non-linearity, such that the statistic is unaffected by the use of proportional or absolute changes in weight. The QCR is 0.915 for the simple glass-box index, compared with 0.697 for the risk model-optimized index. That the QCR is greater than the correlation coefficient for the risk model-optimized index implies that the relationship between the change in weights and ESG scores is indeed non-linear. But even after controlling for this non-linearity, the relationship between the optimized weights and ESG scores is stronger for the simple glass-box index.

The simple glass-box optimization increases the weight assigned to some assets with below target ESG scores because the pivot point of the index is equal to the weighted-average ESG score of the benchmark, which is less than the target. The 148 observations in the top left quadrant exist along the portion of the line in Exhibit 5a between the weighted-average ESG score of the benchmark (64.67) and the target ESG score (71.14). We selected the target value as the reference point for comparing the two methods to minimize bias. But, if we define the quadrants using the weighted-average ESG score of the benchmark index, then the QCR of the simple glass-box index is 1, compared with just 0.73 for the risk model-optimized index.

Finally, we consider the relative magnitudes of the proportional and absolute differences between the constrained and unconstrained weights of the two indices. The absolute changes in weights are often larger for the simple glass-box index. However, the largest redistributions are limited to those assets with the greatest unconstrained benchmark weights. This is due to the condition of proportional redistribution embedded in the glass-box objective function. The impact of this condition is made clear in Exhibit 8a, which shows that the variation in the optimized weights of the simple glass-box index is linearly increasing as a function of the benchmark weights. Since the benchmark is market-cap weighted, this means the greatest redistributions are applied to the largest and most liquid assets.

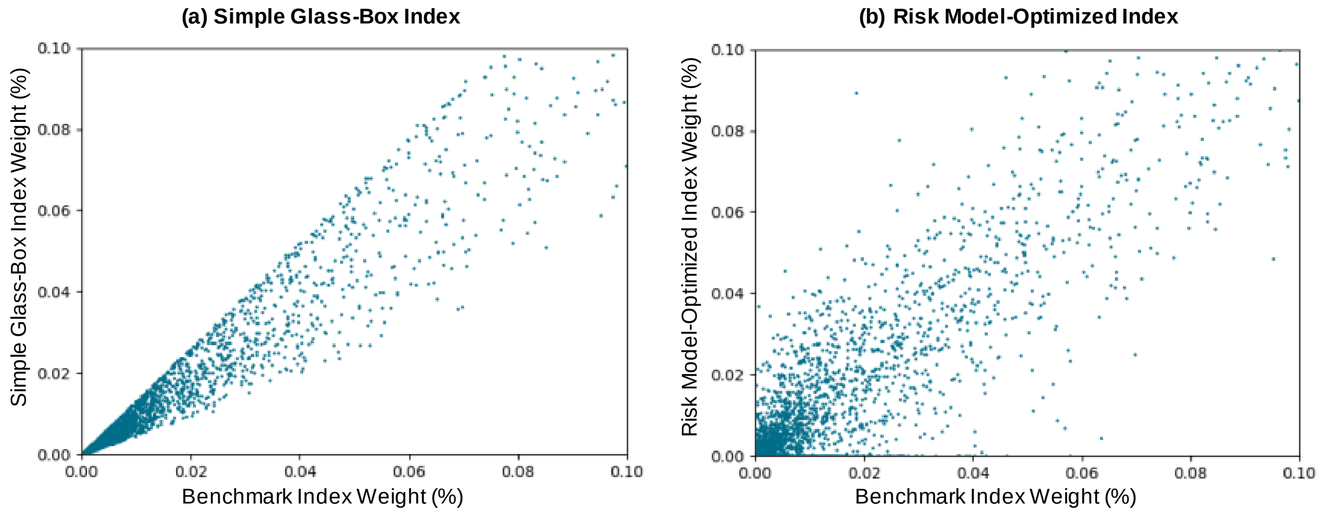
Exhibit 7: Stock Counts for 10 Subsets of the Cross-Section with Proportional Weight Change in Simple Glass-Box and Risk Model-Optimized Indices with ESG Constraint

Year	Proportional Weight Change									
	0-0.1	0.1-0.5	0.5-1	1-2	2-5	5-10	10-25	25-50	50-100	100+
Simple Glass-Box Index										
2016	555	2,074	322	0	0	0	0	0	0	0
2017	686	2,169	71	0	0	0	0	0	0	0
2018	705	2,229	1	0	0	0	0	0	0	0
2019	740	2,525	44	0	0	0	0	0	0	0
2020	746	2,649	0	0	0	0	0	0	0	0
2021	732	2,744	0	0	0	0	0	0	0	0
2022	568	2,653	255	0	0	0	0	0	0	0
Risk Model-Optimized Index										
2016	444	896	1,308	157	104	25	15	2	0	0
2017	497	915	1,203	166	115	17	11	0	2	0
2018	488	923	1,232	174	88	17	11	2	0	0
2019	488	872	1,510	184	136	75	36	3	4	1
2020	575	1,029	1,444	125	113	63	33	11	1	1
2021	494	978	1,600	207	116	52	18	8	2	1
2022	399	830	1,841	198	133	46	16	9	4	0

All indices are hypothetical.
Source: S&P Dow Jones Indices LLC. Data as of May 2022 rebalance. The proportional weight change is equal to the absolute difference between the optimized index weight and the benchmark index weight, divided by the benchmark index weight. Table is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

By contrast, the greatest proportional redistributions are observed in the risk model-optimized index. Exhibit 7 reports the number of instances in which the weight multiplier is between several different thresholds for the last rebalance date of each sample year for both indices. The index produced by the risk model optimization exhibits numerous cases in which the weight multiplier is greater than 10. Furthermore, Exhibit 8b shows that the variation in the risk model-optimized index weights of assets with low benchmark index weights is significantly higher than for the simple glass-box index, suggesting that some of the large proportional redistributions are applied to small, illiquid stocks.

Exhibit 8: Relationship between the ESG-Constrained Simple Glass-Box and Risk Model-Optimized Index Weights and the Benchmark Index Weights



All indices are hypothetical.
Source: S&P Dow Jones Indices LLC. Data as of May 2022 rebalance. The set of observations is limited to assets with constrained and unconstrained weights less than 0.001. Charts are provided for illustrative purposes and reflect hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

Exhibit 9 expands on this point by reporting the number of stocks in 100 subsets of the cross-section with respect to market capitalization and the weight multiplier for the last rebalance date for both indices. The results show that the greatest proportional redistributions imposed by the risk model are indeed concentrated in the smallest and least liquid stocks, suggesting that additional liquidity and stock weight constraints may be required to ensure the efficacy of the risk model-optimized index as a viable investment strategy. These constraints are less likely to be required for the simple glass-box index.

Exhibit 9: Stock Counts for 100 Subsets of the Cross-Section with Respect to Market Cap and Proportional Weight Change for Simple Glass-Box and Risk Model-Optimized Indices with ESG Constraint

Market Cap Decile	Proportional Weight Change									
	0-0.1	0.1-0.5	0.5-1	1-2	2-5	5-10	10-25	25-50	50-100	100+
Simple Glass-Box Index										
1	20	226	68	0	0	0	0	0	0	0
2	27	225	62	0	0	0	0	0	0	0
3	32	231	45	0	0	0	0	0	0	0
4	34	255	22	0	0	0	0	0	0	0
5	38	265	13	0	0	0	0	0	0	0
6	64	248	21	0	0	0	0	0	0	0
7	66	267	12	0	0	0	0	0	0	0
8	81	273	9	0	0	0	0	0	0	0
9	92	317	2	0	0	0	0	0	0	0
10	114	346	1	0	0	0	0	0	0	0
Risk Model-Optimized Index										
1	2	13	238	6	19	14	10	9	4	0
2	5	24	229	21	16	15	4	0	0	0
3	7	25	235	17	15	8	1	0	0	0
4	12	28	241	13	12	4	1	0	0	0
5	19	39	210	22	22	4	0	0	0	0
6	21	59	201	29	22	1	0	0	0	0
7	24	82	173	45	21	0	0	0	0	0
8	32	122	170	33	6	0	0	0	0	0
9	73	209	117	11	0	0	0	0	0	0
10	204	229	27	1	0	0	0	0	0	0

All indices are hypothetical.
Source: S&P Dow Jones Indices LLC. Data as of May 2022 rebalance. The proportional weight change is defined as the constrained weight minus the unconstrained weight, divided by the unconstrained weight. Table is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

In summary, these results provide strong initial support in favor of the simple glass-box optimization as a transparent alternative to risk model-based index optimizations for incorporating a single ESG objective in the construction of an index. The proportional redistributions derived using the simple glass-box optimization are linearly and deterministically related to ESG scores. In other words, the optimized weights are completely explainable in terms of the ESG constraint: companies with an ESG score above (below) the weighted-average ESG score of the benchmark index experience an increase (decrease) in weight relative to their weight in the benchmark index. The simple glass-box index also relies less heavily on extreme positions in small, illiquid assets compared with the index derived using the risk model optimization, such that the former is more likely to represent a tractable investment strategy.

4. Constraining Carbon Intensity

In this section, we expand on the previous analysis by comparing the indices produced by the simple glass-box and risk model optimizations in the presence of a single constraint on weighted-average carbon intensity (WACI). Constraints of this nature are required in a broad range of ESG, net zero and climate transition indices. The carbon intensity of a company is measured as the sum of its Scopes 1 and 2 carbon emissions divided by its enterprise value including cash (EVIC). Scope 1 emissions are from directly emitting sources that are owned or controlled by a company. Scope 2 emissions are from the consumption of purchased electricity, steam or other sources of energy generated upstream from a company's direct operations. The sum of Scope 1 and Scope 2 carbon emissions account for the total emissions directly produced by a company. Carbon emissions data are collected from S&P Global Trucost. The carbon intensity constraint imposes a minimum 75% reduction in WACI relative to the benchmark index. Exhibit 10 reports summary statistics for the simple glass-box and risk model-optimized indices.

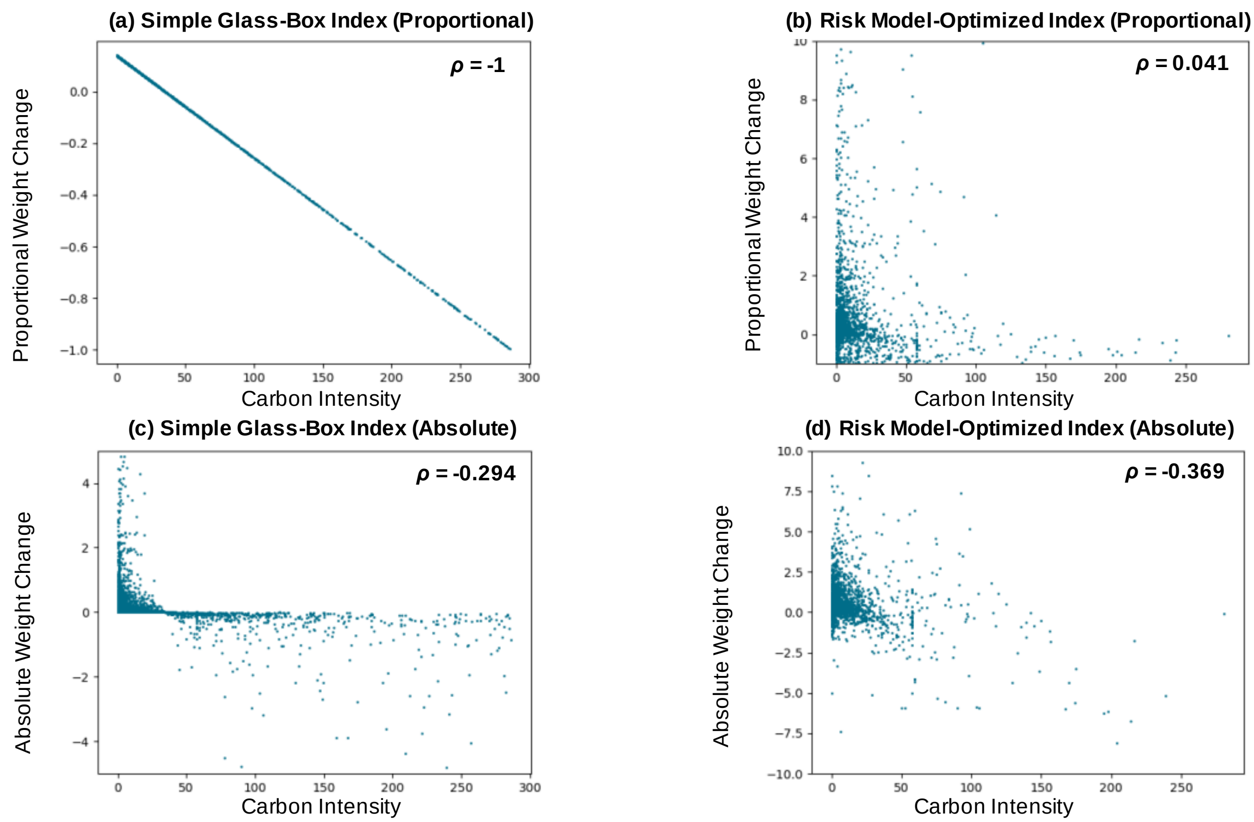
Exhibit 10: Summary Statistics for the Simple Glass-Box and Risk Model-Optimized Indices with Carbon Constraint

Year	Active Share	Stock Count	Weight of Top 10	Effective Number of Shares	WACI	Correlation Weight Carbon
Simple Glass-Box Index						
2016	0.108	2,664	0.099	436.866	25.091	-1
2017	0.105	2,633	0.11	404.431	21.456	-1
2018	0.101	2,627	0.121	356.082	22.204	-1
2019	0.091	2,988	0.131	317.881	20.081	-1
2020	0.073	3,093	0.162	225.036	16.701	-1
2021	0.074	3,142	0.189	185.037	14.085	-1
2022	0.093	3,120	0.171	206.949	14.41	-1
Risk Model-Optimized Index						
2016	0.171	1,931	0.09	416.535	25.088	-0.027
2017	0.156	1,952	0.097	390.631	21.452	-0.015
2018	0.156	2,004	0.109	342.358	22.247	-0.047
2019	0.136	2,235	0.118	308.228	20.081	-0.020
2020	0.120	2,241	0.152	216.400	16.700	-0.028
2021	0.116	2,296	0.173	182.334	14.084	-0.040
2022	0.153	2,024	0.154	201.11	14.386	-0.041

All indices are hypothetical.
Source: S&P Dow Jones Indices LLC. Data as of May 2022 rebalance. Correlation Weight Carbon is the correlation between the proportional changes in index weights and carbon intensities for positively weighted assets. Past performance is no guarantee of future results. Table is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

Consistent with the results in Exhibits 2 and 4, Exhibit 10 shows that the simple glass-box index included a greater number of assets, exhibited significantly lower active share and reported a greater effective number of shares than the risk model-optimized index as of each rebalancing date. Furthermore, the proportional redistributions imposed by the simple glass-box optimization were perfectly negatively correlated with carbon intensity, whereas those derived using the risk model optimization were almost completely uncorrelated with carbon intensity for all dates in the sample. In fact, the estimated correlation coefficients are all statistically indistinguishable from zero for the risk model-optimized index. Exhibit 11 plots the relationship between the proportional and absolute redistributions derived using the simple glass-box and risk model optimizations with carbon intensity for the last rebalance date. The plots reveal the extent to which the weights produced by the risk model optimization are determined by factors independent of the constraints. By contrast, the proportional and absolute redistributions derived using the simple glass-box optimization convey the same level of association as in Section 3: the proportional changes are a linear function of carbon intensity, and the absolute changes are positive (negative) if carbon intensity is less (more) than a well-defined pivot point.

Exhibit 11: Relationship between the Proportional and Absolute Weight Changes and Carbon Intensity for the Simple Glass-Box and Risk Model-Optimized Indices



All indices are hypothetical.
Source: S&P Dow Jones Indices LLC. Data as of May 2022 rebalance. The proportional weight change is defined as the constrained weight minus the unconstrained weight, divided by the unconstrained weight. Charts are provided for illustrative purposes and reflect hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

Exhibit 12 reports the total number of observations in each quadrant of the plots in Exhibit 11, where the quadrants are determined by dividing the vertical axis into values above and below zero and dividing carbon intensities into values above and below the target WACI. The QCR statistics for the simple glass-box and risk model-optimized indices were -0.746 and -0.504, respectively. Again, the simple glass-box index exhibited stronger quadrant association than the risk model-optimized index. But, unlike in Section 3, the simple glass-box index did not convey perfect quadrant association when the quadrants were defined with respect to the WACI of the benchmark. This is because the simple glass-box optimization assigned zero weights to some assets, such that the pivot point became the pro-rata benchmark WACI of the positively weighted assets (see Appendix II).

Exhibit 12: Quadrant Stock Counts for the Simple Glass-Box and Risk Model-Optimized Indices with Carbon Constraint

Simple Glass-Box Index	**Carbon Intensity < Target**	**Carbon Intensity > Target**
Optimized Weight > Benchmark Weight	1,819	441
Benchmark Weight > Optimized Weight	0	1,216
Risk Model-Optimized Index	**Carbon Intensity < Target**	**Carbon Intensity > Target**
Optimized Weight > Benchmark Weight	1,201	245
Benchmark Weight > Optimized Weight	617	1,411

All indices are hypothetical.
Source: S&P Dow Jones Indices LLC. Data as of May 2022 rebalance. Past performance is no guarantee of future results. Table is provided for illustrative purposes and reflect hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

Consistent with Section 3, the index produced by the simple glass-box optimization exhibited larger absolute weight changes compared with the risk model, but these large changes were limited to assets with the greatest benchmark index weights, which are large and highly liquid. Conversely, Exhibits 13 and 14 show that the proportional changes were greatest for the risk model, with the most extreme positions taken in the smallest stocks. For example, the weight multiplier was greater than 10 for the 21 stocks in the first market cap decile, implying that additional liquidity constraints may be necessary.

Exhibit 13: Stock Counts for 100 Subsets of the Cross-Section with Proportional Weight Change for Simple Glass-Box and Risk Model-Optimized Indices with Carbon Constraint

Year	Proportional Weight Change									
	0-0.1	0.1-0.5	0.5-1	1-2	2-5	5-10	10-25	25-50	50-100	100+
Simple Glass-Box Index										
2016	591	1,964	396	0	0	0	0	0	0	0
2017	822	1,700	404	0	0	0	0	0	0	0
2018	718	1,791	426	0	0	0	0	0	0	0
2019	995	1,850	464	0	0	0	0	0	0	0
2020	1,907	1,050	438	0	0	0	0	0	0	0
2021	1,881	1,115	480	0	0	0	0	0	0	0
2022	1,017	1,962	497	0	0	0	0	0	0	0
Risk Model-Optimized Index										
2016	383	795	1,400	180	140	33	15	3	2	0
2017	420	808	1,371	172	103	32	17	3	0	0
2018	463	832	1,296	170	117	35	18	4	0	0
2019	549	878	1,493	171	139	43	26	7	3	0
2020	554	872	1,560	192	135	44	28	7	3	0
2021	559	853	1,618	210	151	52	24	6	3	0
2022	426	728	1,872	188	143	71	38	7	3	0

All indices are hypothetical.
Source: S&P Dow Jones Indices LLC. Data as of May 2022 rebalance. The proportional weight change in weight is defined as the constrained weight minus the unconstrained weight, divided by the unconstrained weight. Table is provided for illustrative purposes and reflect hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

Exhibit 14: Stock Counts for 100 Subsets of the Cross-Section with Respect to Market Cap and Proportional Weight Change for Simple Glass-Box and Risk Model-Optimized Indices with Carbon Constraint

Market Cap Decile	Proportional Weight Change									
	0-0.1	0.1-0.5	0.5-1	1-2	2-5	5-10	10-25	25-50	50-100	100+
Simple Glass-Box Index										
1	87	167	61	0	0	0	0	0	0	0
2	116	157	41	0	0	0	0	0	0	0
3	101	167	40	0	0	0	0	0	0	0
4	92	157	61	0	0	0	0	0	0	0
5	112	145	59	0	0	0	0	0	0	0
6	96	186	51	0	0	0	0	0	0	0
7	114	180	51	0	0	0	0	0	0	0
8	95	221	48	0	0	0	0	0	0	0
9	103	265	42	0	0	0	0	0	0	0
10	101	317	43	0	0	0	0	0	0	0
Risk Model-Optimized Index										
1	5	5	258	6	12	8	11	7	3	0
2	3	22	230	13	11	19	16	0	0	0
3	8	28	219	19	18	13	3	0	0	0
4	9	31	200	23	26	17	5	0	0	0
5	13	42	202	28	24	6	1	0	0	0
6	17	69	195	34	13	4	1	0	0	0
7	23	78	193	28	21	2	0	0	0	0
8	41	117	164	27	11	2	1	0	0	0
9	95	171	129	8	7	0	0	0	0	0
10	212	165	82	2	0	0	0	0	0	0

Source: S&P Dow Jones Indices LLC. Data as of May 2022 rebalance. The proportional weight change in weight is defined as the constrained weight minus the unconstrained weight, divided by the unconstrained weight. Table is provided for illustrative purposes and reflect hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

5. Multiple Constraints

We now consider the case in which the indices are subject to constraints on both ESG score and carbon intensity. Exhibit 15 conveys summary statistics for the two approaches. Consistent with Sections 3 and 4, the simple glass-box index had a higher stock count, a lower active share and greater effective number of shares than the index produced by the risk model for all sample years. However, the correlation between the proportional redistributions and the variables used to define the constraints was no longer perfectly positive or negative for the simple glass-box index. Indeed, this will only be the case with a single constraint or if the variables used to define the constraints are perfectly correlated. This is because correlation is a measure of bivariate association, but the weights are now a function of two variables. Furthermore, as we include more constraints in the optimization problem, the correlation between the weights and each one of the constraints will naturally decrease.

However, we can still prove that the simple glass-box index weights are a linear and deterministic function of the variables used to define the constraints. The only difference is that the solution now includes an extra term for each additional constraint included in the problem. For example, with constraints $(\mathrm{ESG_i})$ on ESG score and carbon intensity $(\mathrm{C_i})$, and if $w_i^* > 0$ for each asset i, the simple glass-box index weights are equal to

$$\mathrm{w_i^*} = \mathrm{w_i} + \lambda_1 \left(\mathrm{ESG_i} - \sum_{\mathrm{j=1}}^{\mathrm{N}} \mathrm{w_j ESG_j} \right) \mathrm{w_i} - \lambda_2 \left(\mathrm{C_i} - \sum_{\mathrm{j=1}}^{\mathrm{N}} \mathrm{w_j C_j} \right) \mathrm{w_i} \tag{4}$$

where $\lambda_1 > 0$ and $\lambda_2 > 0$.[7] The pivot points for ESG score and carbon intensity are the same as in the univariate case, but the magnitude of the coefficients may be different. Specifically, if the covariance between ESG scores and carbon intensity is zero, then λ_1 and λ_2 are the same as when we include a single constraint on ESG score or carbon intensity, respectively, otherwise they are adjusted to account for this covariance. Thus, Equation 5 shows that in the presence of multiple constraints, the proportional redistributions derived using the simple glass-box optimization are linearly dependent on the variables used to define the constraints, such that the simple glass-box index weights remain completely explainable with respect to these variables.

[7] A complete formula for λ_1 and λ_2 is provided in Appendix II.

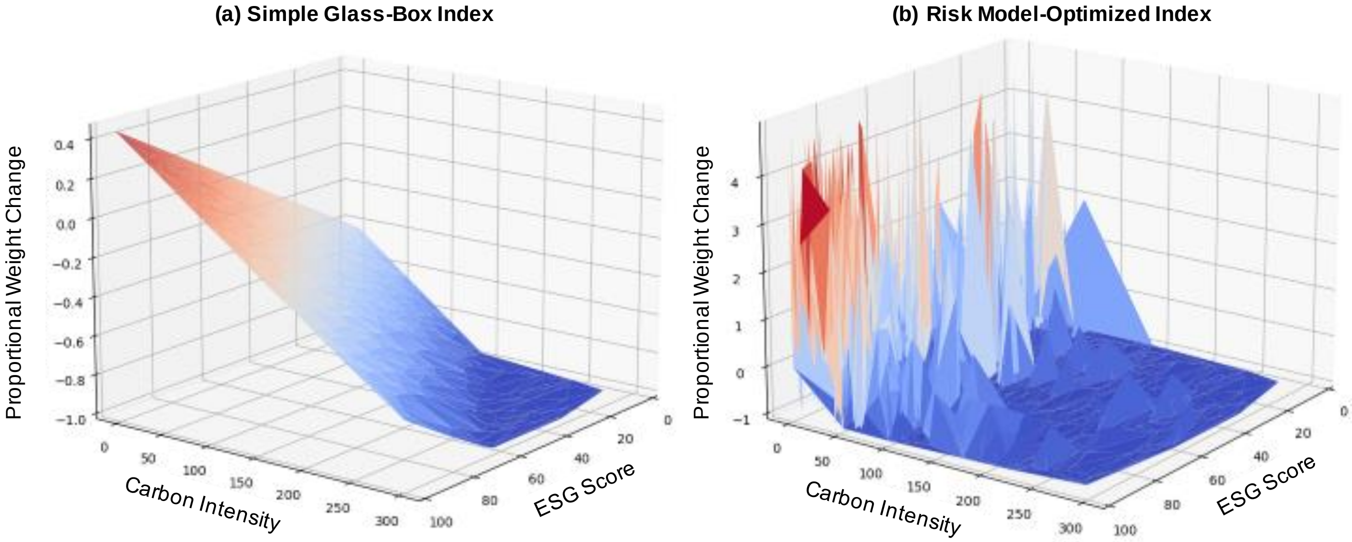
Exhibit 15: Summary Statistics for the Simple Glass-Box and Risk Model-Optimized Indices with ESG and Carbon Constraints

Year	Active Share	Stock Count	Weight of Top 10	Effective Number of Shares	Weighted-Average ESG Score	WACI	Correlation Weight ESG	Correlation Weight Carbon
Simple Glass-Box Index								
2016	0.163	2,617	0.107	436.866	73.356	25.091	0.803	-0.508
2017	0.152	2,607	0.112	404.431	69.614	21.456	0.760	-0.580
2018	0.146	2,593	0.121	356.082	67.678	22.204	0.734	-0.584
2019	0.140	2,955	0.130	317.881	69.493	20.081	0.757	-0.573
2020	0.107	3,072	0.166	225.036	57.576	16.701	0.610	-0.709
2021	0.120	3,118	0.193	185.037	64.431	14.085	0.716	-0.613
2022	0.145	3,093	0.173	206.949	71.13	14.41	0.772	-0.540
Risk Model-Optimized Index								
2016	0.228	1,598	0.091	396.139	73.388	25.084	0.101	-0.029
2017	0.210	1,647	0.099	375.447	69.596	21.452	0.147	-0.014
2018	0.208	1,679	0.107	337.203	67.624	22.247	0.166	-0.028
2019	0.188	1,830	0.118	299.784	69.493	20.081	0.098	-0.023
2020	0.159	1,945	0.156	208.825	57.571	16.700	0.093	-0.006
2021	0.177	1,902	0.178	172.356	64.426	14.084	0.098	-0.027
2022	0.211	1,583	0.154	198.443	71.129	14.386	0.028	-0.048

All indices are hypothetical.
Source: S&P Dow Jones Indices LLC. Data as of May 2022 rebalance. Correlation Weight ESG and Carbon are the correlations between the proportional weight change and ESG score and carbon intensity, respectively. Past performance is no guarantee of future results. Table is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

Despite the limitations of the correlation coefficient in a multivariate setting, we can infer from the observation that the correlation between the proportional redistributions, ESG scores and carbon intensities is higher for the simple glass-box index than for the risk model-optimized index. For example, the correlation between the proportional redistributions and ESG scores ranges from 0.610 to 0.803 for the simple glass-box index, compared with just 0.093 to 0.166 for the risk model-optimized index. This is because the index weights produced by the risk model optimization are a function of several other factors that are included to explain the covariance matrix of returns. Exhibit 16 plots the relationship between the proportional redistributions, ESG scores and carbon intensities for the simple glass-box and risk model-optimized indices. Exhibit 16a depicts the linear relationship described in Equation 5 for the simple glass-box index, while Exhibit 16b conveys almost no association for the risk model-optimized index.

Exhibit 16: Relationship between the Proportional Weight Changes, ESG Scores and Carbon Intensities for the Simple Glass-Box and Risk Model-Optimized Indices



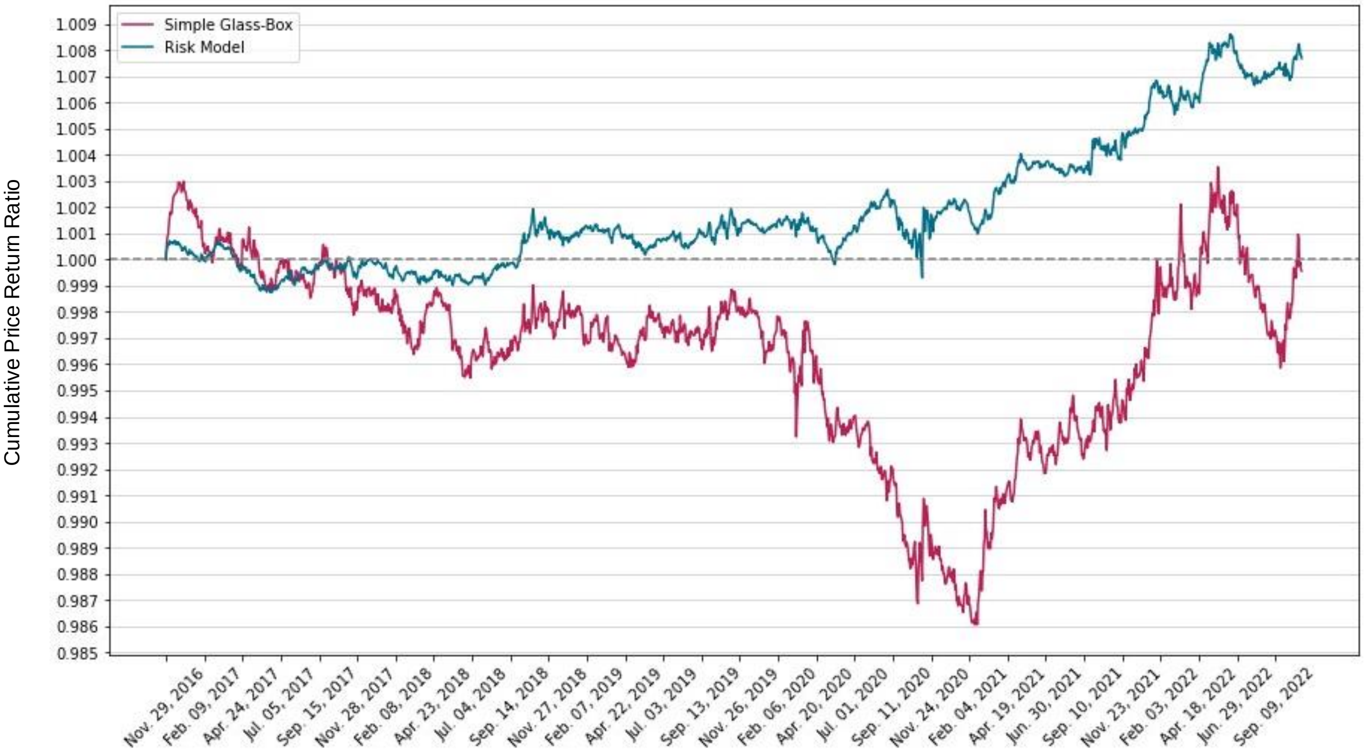
All indices are hypothetical.
Source: S&P Dow Jones Indices LLC. Data as of Mayr 2022 rebalance. Note: the set of observations is limited to include only those assets with a carbon intensity less than 300, and for the risk model-optimized index, proportional weight change less than 5 to aid visual interpretation. Past performance is no guarantee of future results. Charts are provided for illustrative purposes and reflect hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

6. Performance Analysis

This section compares the historical tracking error of the simple glass-box and risk model-optimized indices. Tracking error is calculated as the standard deviation of the difference in returns between the optimized and benchmark indices. The annualized one-, three- and five-year tracking errors, as well as for the full sample period are conveyed in Exhibit 19 for each combination of the constraints. The tracking errors for each sample year are reported in Exhibit 20. Additional performance statistics are provided in Appendix I.

We first consider the case in which the indices are subject to the single weighted-average constraint on ESG score described in Section 2. The cumulative price return ratios of the two approaches are conveyed in Exhibit 17. Exhibits 19 and 20 indicate that the tracking error of the simple glass-box index was approximately twice that of the risk model-optimized index over the full sample period, as well as on an annual basis. Therefore, despite having a higher active share, the index produced by the risk model did achieve a lower tracking error than the simple glass-box index. The same is true with the single constraint on carbon intensity described in Section 4.

Exhibit 17: Historical Back-Tested Cumulative Price Return Ratios for the Simple Glass-Box and Risk Model-Optimized Indices with ESG Constraint



All indices are hypothetical.
Source: S&P Dow Jones Indices LLC. Data from Nov. 29, 2016, to Oct. 31, 2022. The price return ratios are calculated as the ratio of the cumulative price return time series of the optimized indices to that of the benchmark S&P Global LargeMidCap, calculated in USD. Past performance is no guarantee of future results. Chart is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

We now consider the case in which the optimized indices are subject to constraints on both ESG score and carbon intensity. Exhibit 18 conveys the historical cumulative price-to-return ratio for both approaches in this scenario. The results in Exhibits 19 and 20 convey a similar difference between the tracking errors of the simple glass-box and risk model-optimized indices as in the previous scenarios. But interestingly, the annualized tracking error of the simple glass-box index was slightly lower since inception than in the presence of a single constraint on carbon intensity, whereas the tracking error of the risk model-optimized index is slightly higher. This may indicate that the ability of the risk model-optimized index to track the performance of the benchmark is more significantly affected by the inclusion of multiple constraints, such that as the number of constraints increases, the index derived using the glass-box optimization may achieve a lower tracking error than the index derived using the risk model. We leave this as a topic for future research.

Exhibit 18: Historical Back-Tested Cumulative Price Return Ratio for the Simple Glass-Box and Risk Model-Optimized Indices with ESG and Carbon Constraints

All indices are hypothetical.
Source: S&P Dow Jones Indices LLC. Data from Nov. 29, 2016, to Oct. 31, 2022. The price return ratios are calculated as the ratio of the cumulative price return time series of the optimized indices to that of the benchmark S&P Global LargeMidCap benchmark, calculated in USD. Past performance is no guarantee of future results. Chart is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

Exhibit 19: Historical Back-Tested Annualized Tracking Error for the Simple Glass-Box and Risk Model-Optimized Indices with Each Constraint Scenario

Period	Annualized Tracking Error (%)					
	ESG Constraint		Carbon Constraint		ESG and Carbon Constraint	
	Simple Glass-Box Index	Risk Model-Optimized Index	Simple Glass-Box Index	Risk Model-Optimized Index	Simple Glass-Box Index	Risk Model-Optimized Index
One-Year	0.71	0.27	1.31	0.44	1.21	0.44
Three-Year	0.62	0.28	1.06	0.41	1.01	0.44
Five-Year	0.55	0.25	0.92	0.37	0.88	0.40
Since Nov. 30, 2016	0.52	0.23	0.87	0.35	0.84	0.39

All indices are hypothetical.
Source: S&P Dow Jones Indices LLC. Data as of Oct. 31, 2022. Performance based on price return in USD. Past performance is no guarantee of future results. Table is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

Exhibit 20: Historical Back-Tested Annual Tracking Error for the Simple Glass-Box and Risk Model-Optimized Indices with Each Constraint Scenario

Year	Annual Tracking Error (%)					
	ESG Constraint		Carbon Constraint		ESG and Carbon Constraint	
	Simple Glass-Box Index	Risk Model-Optimized Index	Simple Glass-Box Index	Risk Model-Optimized Index	Simple Glass-Box Index	Risk Model-Optimized Index
2017	0.36	0.14	0.48	0.25	0.56	0.29
2018	0.43	0.19	0.78	0.30	0.71	0.37
2019	0.39	0.18	0.51	0.26	0.57	0.31
2020	0.64	0.35	1.07	0.48	1.05	0.52
2021	0.53	0.22	0.75	0.30	0.72	0.32
2022	0.74	0.26	1.40	0.46	1.29	0.46

All indices are hypothetical.
Source: S&P Dow Jones Indices LLC. Data from Jan. 1, 2017, to Oct. 31, 2022. Performance based on price return in USD. Past performance is no guarantee of future results. Table is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

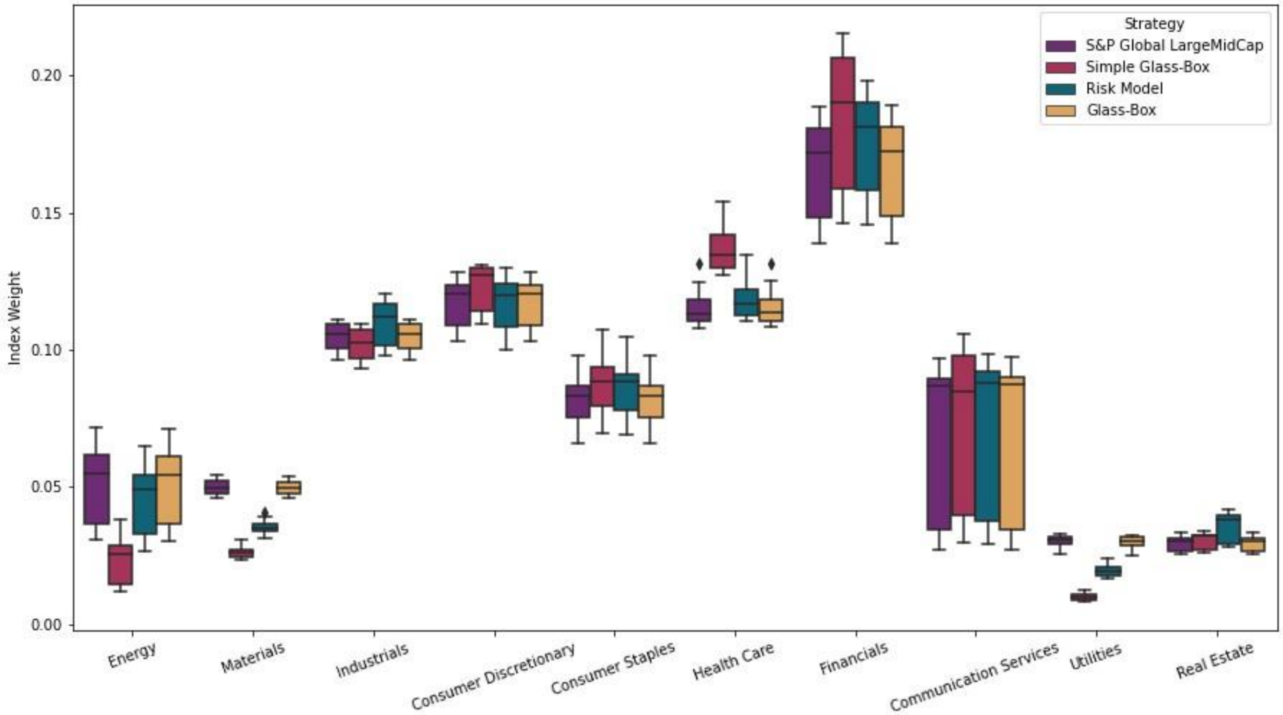
7. Taming Tracking Error

That the tracking error of the simple glass-box index is higher than that of the risk model-optimized index implies that the risk model optimization effectively controls for additional sources of cross-sectional variation in asset returns beyond that which can be achieved by simply minimizing active share. This is to be expected, as was pointed out in Section 3. However, the simple glass-box optimization methodology can be generalized to include additional penalties in the objective function to help control for differences in exposure to certain groups of assets. These group penalties can be constructed with the aim of reducing tracking error. Moreover, the inclusion of group penalties in the simple glass-box objective function makes the proportional change in weight of each asset relative to those within the same group. As noted in the research by Kölbel et al. (2020), this has the effect of incentivizing change among peers.

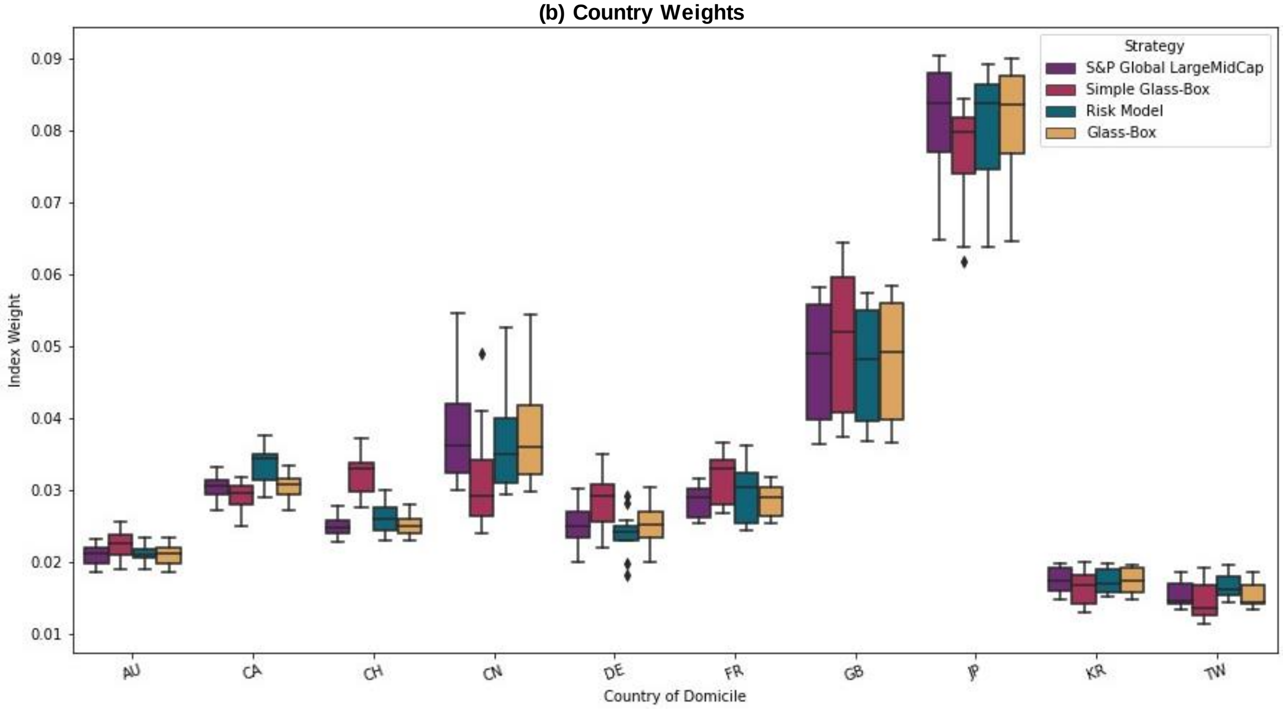
The risk model optimization controls for differences in country and sector weights. To evaluate the impact of these controls, Exhibit 21a plots the GICS® sector weights of the S&P Global LargeMidCap, simple glass-box index and risk model-optimized index subject to both ESG score and WACI constraints. The chart conveys larger active sector weights for the simple glass-box index. In particular, the simple glass-box index underweights Energy, Materials and Utilities and overweights Health Care, Financials and Consumer Staples. Exhibit 21b conveys similar deviations across countries: the simple glass-box index underweights Canada (CA), China (CN) and Japan (JP) and overweights Switzerland (CH), Germany (DE) and France (FR).

Exhibit 21: Distribution of Sector and Country Weights for the S&P Global LargeMidCap and Simple Glass-Box, Risk Model and Glass-Box Indices with ESG and Carbon Constraints

(a) Sector Weights



(b) Country Weights



The simple glass-box, risk model and glass-box indices are hypothetical.
Source: S&P Dow Jones Indices LLC. Data as of May 2022 rebalance. The countries were selected to include the top 10 by average benchmark index weight after removing the U.S., which far outweighs all of the other countries. Charts are provided for illustrative purposes and reflect hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

Therefore, differences in country and sector weights may be a source of tracking error that is not controlled for by the simple glass-box model. S&P DJI's glass-box optimization controls for these differences by including two additional cost terms in the objective function. The glass-box weights are defined as

$$w^* \equiv \underset{w \in S}{\operatorname{argmin}} \left\{ \frac{1}{N} \sum_{i=1}^{N} \left(\frac{(w_i^* - w_i)^2}{w_i} \right) + \frac{1}{M} \sum_{j=1}^{M} \left(\frac{\left(w_j^{s*} - w_j^s\right)^2}{w_j^s} \right) + \frac{1}{K} \sum_{k=1}^{K} \left(\frac{\left(w_k^{c*} - w_k^c\right)^2}{w_k^c} \right) \right\} \tag{4}$$

where w_j^s denotes the weight of sector *j* and w_k^c denotes the weight of country k. In the same way as for the simple glass-box index, division by the sum of the weights in each country and sector ensures proportional redistribution of index weight with respect to these groups. Also, dividing each component in the objective by the number of terms in the summation ensures the contribution of each term is invariant to the number of groups. The index is re-optimized using this objective function with both constraints. Exhibit 22 reports the statistics for the new index, referred to as the glass-box index.

Exhibit 22: Glass-Box Index Summary Statistics with ESG and Carbon Constraints

Year	Active Share	Stock Count	Weight of Top 10	Effective Number of Shares	Weighted-Average ESG Score	WACI	Correlation Weight ESG	Correlation Weight Carbon
2016	0.232	2,373	0.107	436.866	73.356	25.091	0.361	-0.033
2017	0.221	2,296	0.109	404.431	69.614	21.456	0.297	-0.023
2018	0.220	2,350	0.115	356.082	67.678	22.204	0.305	0.006
2019	0.207	2,693	0.124	317.881	69.493	20.081	0.324	-0.050
2020	0.157	2,898	0.164	225.036	57.576	16.701	0.257	-0.124
2021	0.171	2,902	0.186	185.037	64.431	14.085	0.253	-0.131
2022	0.209	2,804	0.165	206.949	71.130	14.410	0.309	-0.130

The index is hypothetical.
Source: S&P Dow Jones Indices LLC. Data as of May 2022 rebalance. Correlation Weight ESG and Carbon are the correlations between the proportional weight change and ESG score and carbon intensity, respectively. Past performance is no guarantee of future results. Table is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

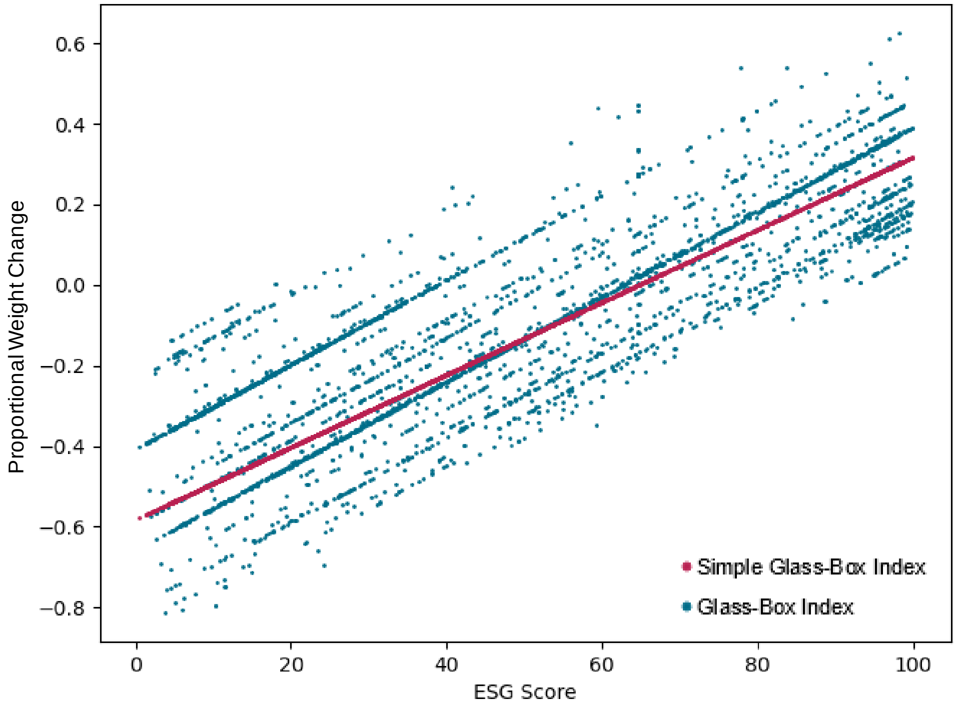
The active share of the glass-box index was greater than that of the simple glass-box index and similar to that of the risk model-optimized index, but the stock count and effective number of shares remained higher for the glass-box index compared with the risk model-optimized index. Consistent with the previous discussion on the limitations of the correlation coefficient as a robust measure of association in the presence of multiple constraints, the correlation between the proportional weight change with ESG score and carbon intensity was lower for the glass-box index compared with the simple glass-box index. In this case, the low correlation is due to the group penalties influencing the glass-box index weights.

However, the index weights produced by the glass-box optimization remain completely explainable in terms of the ESG score and carbon intensity of each asset, even after including group penalties in the objective function. The only difference is that the proportional redistributions are now conditional on both the local (within group) and global (across group) benchmark weighted averages of each variable. To see this, consider the case in which we include a single group penalty and a single constraint on the weighted-average ESG score of the index. Then, the solution for the weight of asset i is given by

$$\mathrm{w_i^*} = \mathrm{w_i} + \delta\left(\mathrm{ESG_i} - \sum_{\mathrm{k=1}}^{\mathrm{N}} \mathrm{w_k ESG_k}\right)\mathrm{w_i} + \gamma\left(\mathrm{ESG_i} - \frac{\sum_{\mathrm{k \in S_j}} \mathrm{w_k ESG_k}}{\sum_{\mathrm{k \in S_j}} \mathrm{w_k}}\right)\mathrm{w_i} \tag{6}$$

where δ and γ are constants, and S_j is the set of assets in group $j \in [1, M]$, with $i \in S_j$. A detailed formula for δ and γ is provided in Appendix II. Note that this solution is conditional on the assumption that $w_i^* > 0$ for each asset i. When the number of groups is equal to one, or every asset exists in its own group (i.e., $M = N$), then the solution reduces to that of the simple glass-box optimization in Equation 2. The first and second terms on the right-hand side are similar to those in the solution for the simple glass-box index, while the final term is unique to the case in which we include a group penalty in the objective function. This term captures the difference between the ESG score of asset i and the pro-rata weighted-average ESG score of the assets in the same group. Hence, the optimized weight of asset i now depends on how its ESG score compares to the weighted-average ESG scores of both the benchmark and the group. If the weighted-average ESG score is different across groups, then the proportional redistributions exist along a set of parallel lines when plotted against ESG scores, where each line corresponds to a different group. Exhibit 23 makes this clear as it plots the proportional redistributions of the glass-box index with a single group penalty on country of domicile against ESG scores: the blue dots correspond to the glass-box index with the country penalty and the red dots correspond to the simple glass-box index. The blue dots trace out a set of parallel lines, where each line corresponds to a different country. Exhibit 23 also shows that by including a country penalty, the gradient of the tilt required to satisfy the constraint increases.

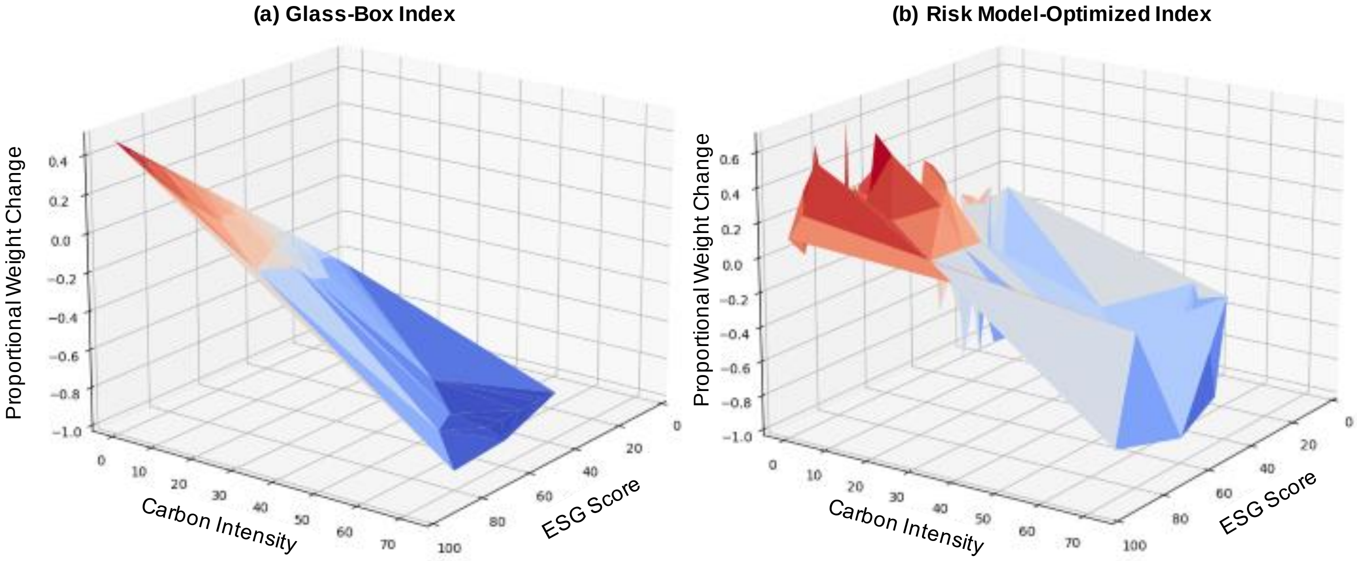
Exhibit 23: Relationship between the Proportional Weight Change and ESG Score for the Simple Glass-Box and Glass-Box Indices with ESG and Carbon Constraints



All indices are hypothetical.
Source: S&P Dow Jones Indices LLC. Data as of May 2022 rebalance. Past performance is no guarantee of future results. Chart is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

Most importantly, Exhibit 23 shows that for each country, there is a linear relationship between the proportional redistributions and ESG scores. This remains true in the case of multiple group penalties and constraints. For example, Exhibit 24a plots the proportional redistributions implied by the optimized weights derived using the glass-box objective function in Equation 6 against ESG scores and carbon intensities as of the May 2022 rebalance. The plot conveys the same linear relationship as that portrayed in Exhibit 16, consistent with the proportional redistributions being linear in the constraints within each intersection of groups. By contrast, Exhibit 24b conveys a far weaker relationship between the constraints and the redistributions derived using the risk model optimization.

Exhibit 24: Relationship between the Proportional Weight Change, ESG Score and Carbon Intensity for the U.S. Information Technology Sector



All indices are hypothetical.
Source: S&P Dow Jones Indices LLC. Data as of May 2022 rebalance. Past performance is no guarantee of future results. Charts are provided for illustrative purposes and reflect hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

The sector and country weights of the glass-box index are included in Exhibits 21a and 21b, respectively. Exhibit 24 shows that the sector and country weights of the glass-box index were almost identical to those of the S&P Global LargeMidCap, implying that the sector and country penalties were effective in reducing the magnitude of active sector and country bets. Exhibit 26 conveys the historical cumulative price return ratios for the simple glass-box, risk model and glass-box indices for the full sample period. The annualized multi-year and annual tracking errors are reported in Exhibits 27 and 28, respectively. The results convey a significant improvement in the tracking error of the glass-box index after including the country and sector penalties in the objective function. For example, the annualized tracking error of the glass-box index was 0.56% since inception, compared with 0.84% for the simple glass-box index. This implies that the country and sector classifications were useful variables for explaining the cross-section of returns. However, the tracking error of the glass-box index was still greater than that of the risk model-optimized index, suggesting that there are additional factors included in the risk model that are important for explaining the cross-sectional variation in returns. One option is to increase the number group penalties included in the glass-box objective function. However, this will increase the magnitude of the proportional redistributions required within each group, which could increase active share, reduce stock count and lead to higher index turnover.

Exhibit 25: Historical Cumulative Price Return Ratios for the Simple Glass-Box, Risk Model and Glass-Box Indices with ESG and Carbon Constraints

All indices are hypothetical.
Source: S&P Dow Jones Indices LLC. Data from Nov. 29, 2016, to Oct. 31, 2022. The price return ratios are calculated as the ratio of the cumulative price return time series of the optimized indices and the benchmark S&P Global LargeMidCap, calculated in USD. Past performance is no guarantee of future results. Chart is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

Exhibit 26: Historical Back-Tested Annualized Tracking Error for the Simple Glass-Box, Risk Model and Glass-Box Indices with ESG and Carbon Constraints

Period	Annualized Turnover (%)		
	Simple Glass-Box Index	Risk Model-Optimized Index	Glass-Box Index
One-Year	1.21	0.44	0.66
Three-Year	1.01	0.44	0.65
Five-Year	0.88	0.40	0.59
Since Nov. 30, 2016	0.84	0.39	0.56

All indices are hypothetical.
Source: S&P Dow Jones Indices LLC. Data as of Oct. 31, 2022. Past performance is no guarantee of future results. Table is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

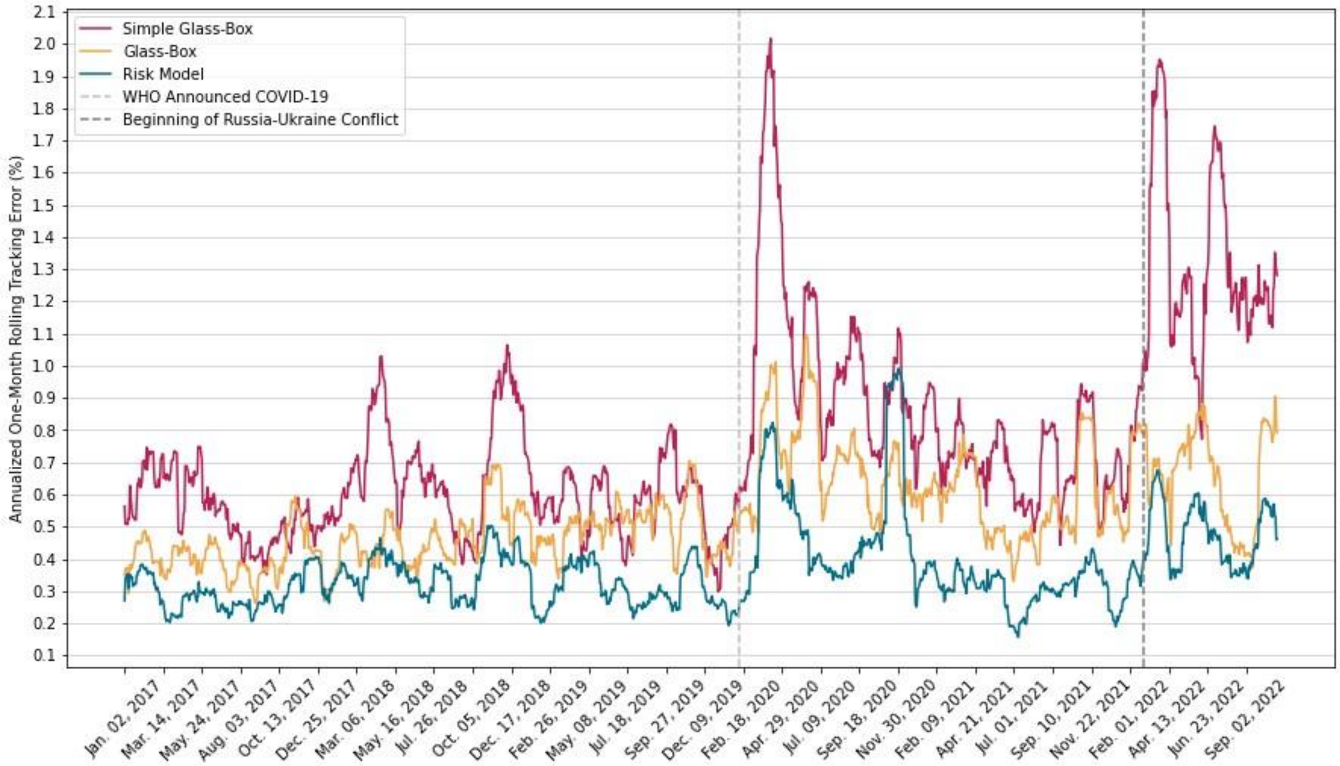
Exhibit 27: Historical Back-Tested Annual Tracking Error for the Simple Glass-Box, Risk Model and Glass-Box Indices with ESG and Carbon Constraints

Year	Turnover (%)		
	Simple Glass-Box Index	Risk Model-Optimized Index	Glass-Box Index
2017	0.56	0.29	0.41
2018	0.71	0.37	0.48
2019	0.57	0.31	0.50
2020	1.05	0.52	0.70
2021	0.72	0.32	0.58
2022	1.29	0.46	0.68

All indices are hypothetical.
Source: S&P Dow Jones Indices LLC. Data as of Oct. 31, 2022. Past performance is no guarantee of future results. Table is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

Next, we investigate the time-series properties of the tracking errors of the optimized indices. Exhibit 29 plots the one-month rolling tracking error for each index and allow us to draw three main conclusions. First, the tracking error of the glass-box index was almost always less than the simple glass-box index. Second, the inclusion of country and sector terms yielded the greatest improvement in tracking error during periods of high macroeconomic uncertainty. The vertical dotted lines in the chart represent the date the World Health Organization (WHO) announced the name of the COVID-19 virus and the date the Russia-Ukraine conflict started. Shortly after these events, the tracking error of the simple glass-box index spiked to roughly 2%, while the tracking error of glass-box index remained relatively stable. Finally, the plot shows that there were occasional periods in which the tracking error of the glass-box index was less than that of the risk model-optimized index (e.g., in October 2020). The tracking error of the risk model-optimized index increasing so significantly during this period and after the other two macroeconomic shocks suggests that the additional complexity involved in the estimation of the risk model made the resulting index more sensitive to underlying regime changes and structural breaks in the data. This sensitivity may also be a consequence of the risk model overfitting noise in-sample. Financial time series are noisy, which increases the chance of overfitting, and straightforward models like the glass-box optimization can often achieve more robust out-of-sample performance than highly parameterized models like those that underpin most risk model optimizations.

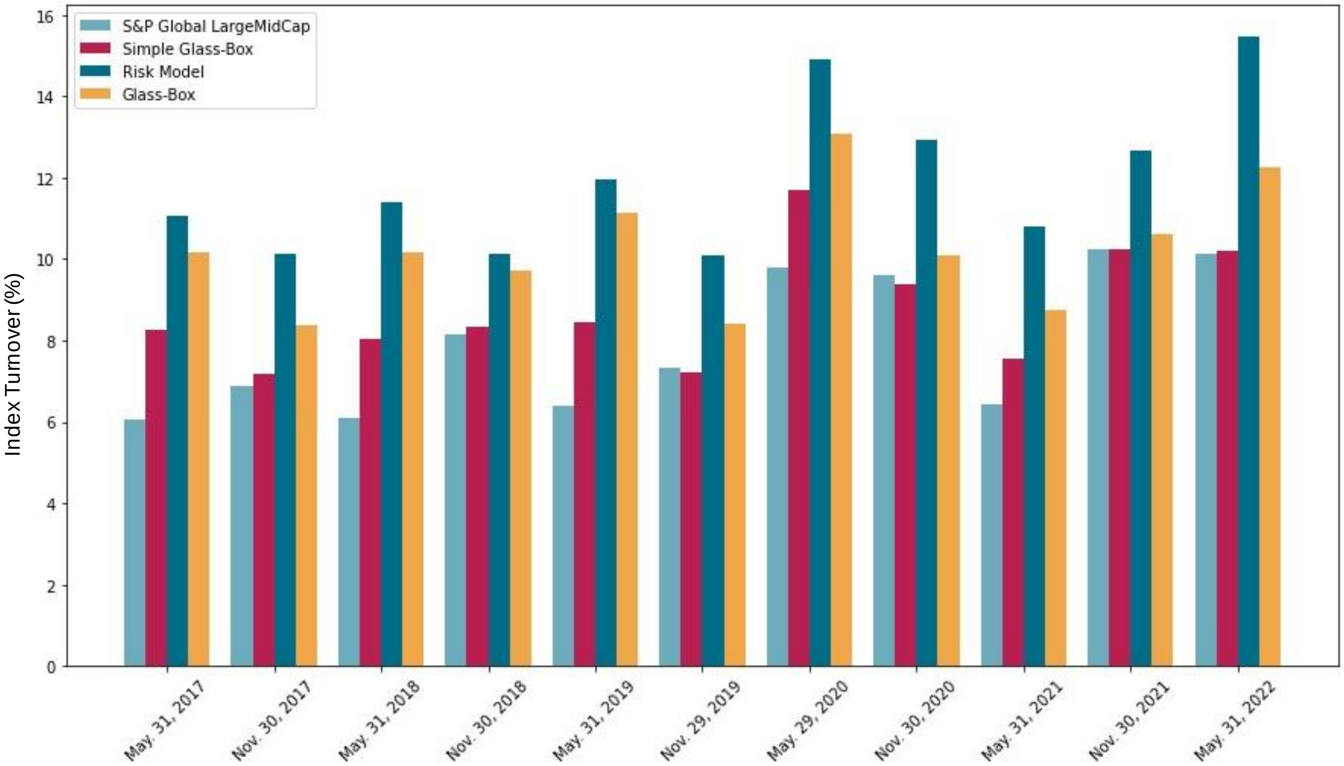
Exhibit 28: Annualized One-Month Rolling Tracking Error for the Simple Glass-Box, Risk Model and Glass-Box Indices with ESG and Carbon Constraints



All indices are hypothetical.
Source: S&P Dow Jones Indices LLC. Data from Jan. 1, 2017, to Oct. 31, 2022. Past performance is no guarantee of future results. Chart is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

Finally, we consider the turnover of the simple glass-box, risk model and glass-box indices. The turnover for each strategy is conveyed in Exhibit 29 as of each rebalancing date, along with the turnover of the benchmark index. Exhibit 29 shows that the turnover was the lowest for the simple glass-box index, and that controlling for differences in country and sector weights increased index turnover for the glass-box index, though it still remained well below that of the risk model-optimized index on all rebalancing dates. On average, the turnover of the simple glass-box and glass-box indices was approximately 3.2% and 1.7% lower than that of the risk model-optimized index, respectively, reflecting an average proportional improvement of 26% and 14%, respectively. The turnover of the glass-box and risk model-optimized indices can be constrained explicitly, but this may come at the cost of higher tracking error and lower liquidity and may also lead to significant path dependence in the selected set of constituents.

Exhibit 29: Index Turnover for the Simple Glass-Box, Risk Model and Glass-Box Indices with ESG and Carbon Constraints



Rebalance Date	Index Turnover (%)			
	S&P Global LargeMidCap	Simple Glass-Box Index	Risk Model-Optimized Index	Glass-Box Index
May 31, 2017	6.05	8.26	11.06	10.15
Nov. 30, 2017	6.86	7.16	10.13	8.37
May 31, 2018	6.11	8.05	11.38	10.18
Nov. 30, 2018	8.16	8.33	10.14	9.71
May 31, 2019	6.40	8.45	11.97	11.14
Nov. 29, 2019	7.33	7.22	10.08	8.40
May 29, 2020	9.79	11.71	14.91	13.07
Nov. 30, 2020	9.61	9.39	12.94	10.10
May 31, 2021	6.44	7.54	10.79	8.76
Nov. 30, 2021	10.23	10.24	12.66	10.63
May 31, 2022	10.14	10.20	15.47	12.25

All indices are hypothetical.
Source: S&P Dow Jones Indices LLC. Data as of May 2022 rebalance. Turnover is calculated as one-half the sum of the absolute differences between the index weights between rebalances. Note: this represents an approximation of the true index turnover that would b e experienced in real time. Past performance is no guarantee of future results. Table is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

8. Conclusion

This paper has investigated two alternative approaches for incorporating multiple sustainability-related objectives in an index-based framework. The two approaches are a representative risk model-based index optimization that minimizes expected tracking error by matching the risk profile of the optimized and benchmark indices with respect to a risk model for returns, and S&P DJI's glass-box optimization, which minimizes active share subject to the condition of proportional redistribution.

The glass-box and risk model-optimized indices were compared with respect to their ability to improve the ESG and carbon characteristics of a global equity benchmark in three different scenarios, where each scenario differed with respect to the set of constraints included in the optimization problem. In all three scenarios, the weights produced by the glass-box optimization demonstrated a high degree of explainability with respect to the ESG score and carbon intensity of each asset, producing a broad shift in exposure away from firms with low ESG scores and high carbon intensity and toward firms with high ESG scores and low carbon intensity. By contrast, the relationship between the weights and the constraints was far weaker for the indices produced by the risk model optimization, reflecting the strong influence of the additional factors included in the risk model on the optimized index weights. The glass-box indices also exhibited greater diversification and lower active share than the risk model optimization and relied less heavily on extreme positions in the smallest and least liquid stocks.

Historical back-tests revealed that risk model optimization achieved lower tracking errors than simple glass-box optimization, indicating that at least some of the additional factors included in the risk model are important for explaining returns. Subsequent analysis showed that the tracking error of the simple glass-box index could be significantly reduced by including country and sector group penalties in the objective function. These penalties reduced the magnitude of active country and sector bets, with the combined effect of reducing the tracking error of the glass-box index by one-third. The advantage of the glass-box framework is that including group penalties did not lead to any significant loss of interpretability or explainability: the linear relationship between the proportional redistributions and the constraints was always preserved within each intersection of groups, thereby ensuring that the individual stock redistributions produced by the glass-box optimization were completely explainable in terms of the company characteristics used to define the constraints.

In summary, these results provide strong support in favor of the glass-box optimization as a transparent alternative to risk model-based optimizations for constructing sustainability indices. In keeping with research by Kölbel et al. (2020), glass-box indices may be better positioned to take advantage of the capital allocation and investment benchmarking impact mechanisms that have been shown to play a vital role in transmitting investor preferences into changes in companies' sustainability practices. However, the applicability of the glass-box optimization extends beyond just sustainability and ESG-oriented indices. More generally, the glass-box optimization could be well suited to the construction of any constrained index where the traceability of the constraints on the optimized index weights is of paramount importance.

Appendix I: Performance Statistics

Exhibit 30: Historical Back-Tested Performance of Simple Glass-Box and Risk Model-Optimized Indices with ESG Constraint

Period	S&P Global LargeMidCap	Simple Glass-Box Index	Risk Model-Optimized Index
Annualized Return (%)			
One-Year	-21.78	-21.35	-21.50
Three-Year	2.86	2.91	3.09
Five-Year	3.20	3.20	3.37
Since Nov. 30, 2016	5.96	5.94	6.09
Annualized Volatility (%)			
One-Year	18.17	18.00	18.15
Three-Year	19.51	19.53	19.51
Five-Year	16.61	16.61	16.62
Since Nov. 30, 2016	15.44	15.45	15.45
Risk-Adjusted Return			
One-Year	-1.20	-1.19	-1.18
Three-Year	0.15	0.15	0.16
Five-Year	0.19	0.19	0.20
Since Nov. 30, 2016	0.39	0.38	0.39
Tracking Error (%)			
One-Year	-	0.71	0.27
Three-Year	-	0.62	0.28
Five-Year	-	0.55	0.25
Since Nov. 30, 2016	-	0.52	0.23

The simple glass-box and risk model-optimized indices are hypothetical.
Source: S&P Dow Jones Indices LLC. Data as of Oct. 31, 2022. Past performance is no guarantee of future results. Table is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

Exhibit 31: Historical Back-Tested Performance for the Simple Glass-Box and Risk Model-Optimized Indices with Carbon Constraint

Period	S&P Global LargeMidCap	Simple Glass-Box Index	Risk Model-Optimized Index
Annualized Return (%)			
One-Year	-21.78	-22.92	-21.81
Three-Year	2.86	3.13	2.99
Five-Year	3.20	3.67	3.40
Since Nov. 30, 2016	5.96	6.42	6.06
Annualized Volatility (%)			
One-Year	18.17	18.73	18.19
Three-Year	19.51	19.68	19.46
Five-Year	16.61	16.79	16.57
Since Nov. 30, 2016	15.44	15.61	15.41
Risk-Adjusted Return			
One-Year	-1.20	-1.22	-1.20
Three-Year	0.15	0.16	0.15
Five-Year	0.19	0.22	0.21
Since Nov. 30, 2016	0.39	0.41	0.39
Tracking Error (%)			
One-Year	-	1.31	0.44
Three-Year	-	1.06	0.41
Five-Year	-	0.92	0.37
Since Nov. 30, 2016	-	0.87	0.35

The simple glass-box and risk model-optimized indices are hypothetical.
Source: S&P Dow Jones Indices LLC. Data as of Oct. 31, 2022. Past performance is no guarantee of future results. Table is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

Exhibit 32: Historical Back-Tested Performance for the Simple Glass-Box, Risk Model-Optimized and Glass-Box Indices with ESG and Carbon Constraints

Period	S&P Global LargeMidCap	Simple Glass-Box Index	Risk Model-Optimized Index	Glass-Box Index
Annualized Return (%)				
One-Year	-21.78	-22.60	-21.74	-21.76
Three-Year	2.86	3.09	3.02	2.94
Five-Year	3.20	3.55	3.36	3.24
Since Nov. 30, 2016	5.96	6.32	6.01	5.85
Annualized Volatility (%)				
One-Year	18.17	18.54	18.15	18.06
Three-Year	19.51	19.66	19.46	19.42
Five-Year	16.61	16.76	16.57	16.55
Since Nov. 30, 2016	15.44	15.59	15.41	15.39
Risk-Adjusted Return				
One-Year	-1.20	-1.22	-1.20	-1.20
Three-Year	0.15	0.16	0.16	0.15
Five-Year	0.19	0.21	0.20	0.20
Since Nov. 30, 2016	0.39	0.41	0.39	0.38
Tracking Error (%)				
One-Year	-	1.21	0.44	0.66
Three-Year	-	1.01	0.41	0.65
Five-Year	-	0.88	0.40	0.59
Since Nov. 30, 2016	-	0.84	0.39	0.56

The simple glass-box, risk model-optimized and glass-box indices are hypothetical.
Source: S&P Dow Jones Indices LLC. Data as of Oct. 31, 2022. Past performance is no guarantee of future results. Table is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance.

Appendix II: Solving the Glass-Box Model

As shown in Section 5 of this paper, the simple glass-box and glass-box indices show linear relations between overweight constituents relative to the benchmark index and the values of the variables considered in the constraints. These relationships can be mathematically formulated and proven. In this appendix, we will provide some of the final formulations of these linear relationships without proofs. A mathematical proof of these results is outside the scope of this paper.

Simple Glass-Box Index with One Constraint

The simple glass-box optimization with a single weighted-average constraint on the value of an arbitrary variable Z is

$$\min_{\mathbf{w}\geq\mathbf{0}} \frac{1}{2}\sum_{\mathrm{i}=1}^{\mathrm{N}}\left(\frac{(\mathrm{w}_\mathrm{i}^* - \mathrm{w}_\mathrm{i})^2}{\mathrm{w}_\mathrm{i}}\right) \ \text{ s. t.}$$

$$\sum_{i=1}^{N} w_i^* Z_i \geq z \ , \qquad \sum_{i=1}^{N} w_i^* = 1, \qquad \textit{and } w_i^* \geq 0 \ \forall \ i.$$

In this case, the simple glass-box index weights of positively weighted assets are

$$\mathrm{w}_\mathrm{i}^* = \widetilde{\mathrm{w}}_\mathrm{i} + \lambda\left(\mathrm{Z}_\mathrm{i} - \sum_{\mathrm{j}\in\mathcal{P}} \widetilde{\mathrm{w}}_\mathrm{j}\mathrm{Z}_\mathrm{j}\right)\widetilde{\mathrm{w}}_\mathrm{i}$$

with

$$\lambda = \frac{\mathrm{z} - \sum_{\mathrm{j}\in\mathcal{P}} \widetilde{\mathrm{w}}_\mathrm{j}\mathrm{Z}_\mathrm{j}}{\left(\left(\sum_{\mathrm{j}\in\mathcal{P}} \widetilde{\mathrm{w}}_\mathrm{j}\mathrm{Z}_\mathrm{j}^2\right) - \left(\sum_{\mathrm{j}\in\mathcal{P}} \widetilde{\mathrm{w}}_\mathrm{j}\mathrm{Z}_\mathrm{j}\right)^2\right)}$$

where, $\mathcal{P}$ is the set of positively weighted assets, and $\widetilde{w}_i = \frac{w_i}{\sum_{i\in\mathcal{P}} w_i}$ is the pro-rata benchmark index weight of asset $i \in \mathcal{P}$.

Simple Glass-Box Index with Multiple Constraints

The solution for the simple glass-box index weights with constraints on the weighted-average values of two variables X and Y, when all constituents are assigned positive weight, is

$$\mathrm{w}_\mathrm{i}^* = \mathrm{w}_\mathrm{i} + \lambda_\mathrm{X}\left(\mathrm{X}_\mathrm{i} - \sum_{\mathrm{j}=1}^{\mathrm{N}} \mathrm{w}_\mathrm{j}\mathrm{X}_\mathrm{j}\right)\mathrm{w}_\mathrm{i} + \lambda_\mathrm{Y}\left(\mathrm{Y}_\mathrm{i} - \sum_{\mathrm{j}=1}^{\mathrm{N}} \mathrm{w}_\mathrm{j}\mathrm{Y}_\mathrm{j}\right)\mathrm{w}_\mathrm{i} \tag{A2}$$

where

$$\lambda_X = \frac{\left(x - \sum_{j=1}^{N} w_j X_j\right)\left(\sum_{j=1}^{N} w_j Y_j^2 - \left(\sum_{j=1}^{N} w_j Y_j\right)^2\right) - \left(y - \sum_{j=1}^{N} w_j Y_j\right)\left(\sum_{j=1}^{N} w_j X_j Y_j - \sum_{j=1}^{N} w_j X_j \sum_{j=1}^{N} w_j Y_j\right)}{\left(\sum_{j=1}^{N} w_j X_j^2 - \left(\sum_{j=1}^{N} w_j X_j\right)^2\right)\left(\sum_{j=1}^{N} w_j Y_j^2 - \left(\sum_{j=1}^{N} w_j Y_j\right)^2\right) - \left(\sum_{j=1}^{N} w_j X_j Y_j - \sum_{j=1}^{N} w_j X_j \sum_{j=1}^{N} w_j Y_j\right)^2} \quad \text{(A3)}$$

$$\lambda_Y = \frac{\left(y - \sum_{j=1}^{N} w_j Y_j\right)\left(\sum_{j=1}^{N} w_j X_j^2 - \left(\sum_{j=1}^{N} w_j X_j\right)^2\right) - \left(x - \sum_{j=1}^{N} w_j X_j\right)\left(\sum_{j=1}^{N} w_j X_j Y_j - \sum_{j=1}^{N} w_j X_j \sum_{j=1}^{N} w_j Y_j\right)}{\left(\sum_{j=1}^{N} w_j X_j^2 - \left(\sum_{j=1}^{N} w_j X_j\right)^2\right)\left(\sum_{j=1}^{N} w_j Y_j^2 - \left(\sum_{j=1}^{N} w_j Y_j\right)^2\right) - \left(\sum_{j=1}^{N} w_j X_j Y_j - \sum_{j=1}^{N} w_j X_j \sum_{j=1}^{N} w_j Y_j\right)^2} \quad \text{(A4)}$$

Letting $\mathcal{P}$ be the set of positively weighted assets, the general solution can be obtained from Equations A1-A3 by replacing the benchmark weights w_i by the pro-rata benchmark index weight of asset $\widetilde{w}_i = \frac{w_i}{\sum_{j \in \mathcal{P}} w_j}$ for every $i \in \mathcal{P}$ and $\widetilde{w}_i = 0$ for each $i \notin \mathcal{P}$.

Glass-Box Index with Group Penalties

The solution for the glass-box index weights with P group penalties and K weighted-average constraints is

$$w^* = w - \Omega^{-1} A' (A \Omega^{-1} A')^{-1} (Aw - b)$$

where, w^* and w are the constrained and unconstrained weight vectors, A is a matrix of constraint variables, b is the vector of target values and Ω is a square matrix. With a single group penalty and a single constraint on the value of an arbitrary variable Z, the simple glass-box index weight of asset i in group k is given by

$$w_i^* = w_i + \delta\left(Z_i - \sum_{j=1}^{N} w_j Z_j\right) w_i + \gamma\left(Z_i - \frac{\sum_{j \in S_k} w_j Z_j}{\sum_{j \in S_k} w_j}\right) w_i$$

where

$$\delta = \frac{\left(z - \sum_{j=1}^{N} w_j Z_j\right)}{\left(\sum_{j=1}^{N} w_j Z_j^2 - \left(\sum_{j=1}^{N} w_j Z_j\right)^2\right) + \frac{N}{M} \sum_{m=1}^{M} \sum_{j \in S_m} w_j \left(\sum_{n \in S_m} \widetilde{w}_n Z_n^2 - \left(\sum_{n \in S_m} \widetilde{w}_n Z_n\right)^2\right)}$$

$$\gamma = \frac{\left(z - \sum_{j=1}^{N} w_j Z_j\right)}{\frac{M}{N}\left(\sum_{j=1}^{N} w_j Z_j^2 - \left(\sum_{j=1}^{N} w_j Z_j\right)^2\right) + \sum_{m=1}^{M} \sum_{j \in S_m} w_j \left(\sum_{n \in S_m} \widetilde{w}_n Z_n^2 - \left(\sum_{n \in S_m} \widetilde{w}_n Z_n\right)^2\right)}$$

and S_m is the set of assets in group $m \in [1, M]$.

Performance Disclosure/Back-Tested Data

All information presented prior to an index's Launch Date is hypothetical (back-tested), not actual performance. The back-test calculations are based on the same methodology that was in effect on the index Launch Date. However, when creating back-tested history for periods of market anomalies or other periods that do not reflect the general current market environment, index methodology rules may be relaxed to capture a large enough universe of securities to simulate the target market the index is designed to measure or strategy the index is designed to capture. For example, market capitalization and liquidity thresholds may be reduced. Complete index methodology details are available at www.spglobal.com/spdji. Past performance of the Index is not an indication of future results. Back-tested performance reflects application of an index methodology and selection of index constituents with the benefit of hindsight and knowledge of factors that may have positively affected its performance, cannot account for all financial risk that may affect results and may be considered to reflect survivor/look ahead bias. Actual returns may differ significantly from, and be lower than, back-tested returns. Past performance is not an indication or guarantee of future results. Please refer to the methodology for the Index for more details about the index, including the manner in which it is rebalanced, the timing of such rebalancing, criteria for additions and deletions, as well as all index calculations. Back-tested performance is for use with institutions only; not for use with retail investors.

S&P Dow Jones Indices defines various dates to assist our clients in providing transparency. The First Value Date is the first day for which there is a calculated value (either live or back-tested) for a given index. The Base Date is the date at which the index is set to a fixed value for calculation purposes. The Launch Date designates the date when the values of an index are first considered live: index values provided for any date or time period prior to the index's Launch Date are considered back-tested. S&P Dow Jones Indices defines the Launch Date as the date by which the values of an index are known to have been released to the public, for example via the company's public website or its data feed to external parties. For Dow Jones-branded indices introduced prior to May 31, 2013, the Launch Date (which prior to May 31, 2013, was termed "Date of introduction") is set at a date upon which no further changes were permitted to be made to the index methodology, but that may have been prior to the Index's public release date.

Typically, when S&P DJI creates back-tested index data, S&P DJI uses actual historical constituent-level data (e.g., historical price, market capitalization, and corporate action data) in its calculations. As ESG investing is still in early stages of development, certain datapoints used to calculate S&P DJI's ESG indices may not be available for the entire desired period of back-tested history. The same data availability issue could be true for other indices as well. In cases when actual data is not available for all relevant historical periods, S&P DJI may employ a process of using "Backward Data Assumption" (or pulling back) of ESG data for the calculation of back-tested historical performance. "Backward Data Assumption" is a process that applies the earliest actual live data point available for an index constituent company to all prior historical instances in the index performance. For example, Backward Data Assumption inherently assumes that companies currently not involved in a specific business activity (also known as "product involvement") were never involved historically and similarly also assumes that companies currently involved in a specific business activity were involved historically too. The Backward Data Assumption allows the hypothetical back-test to be extended over more historical years than would be feasible using only actual data. For more information on "Backward Data Assumption" please refer to the FAQ. The methodology and factsheets of any index that employs backward assumption in the back-tested history will explicitly state so. The methodology will include an Appendix with a table setting forth the specific data points and relevant time period for which backward projected data was used.

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