

## A Primer On Tipping Points

Previewing our upcoming macro-environmental research

May 01, 2024

This report does not constitute a rating action

### What Are Tipping Points?

Tipping points occupy a central, rising, profile in the sustainability lexicon. Because of the increasing focus on the effects of climate change and the possibility that they could be irreversible, this rising profile is important to understand.

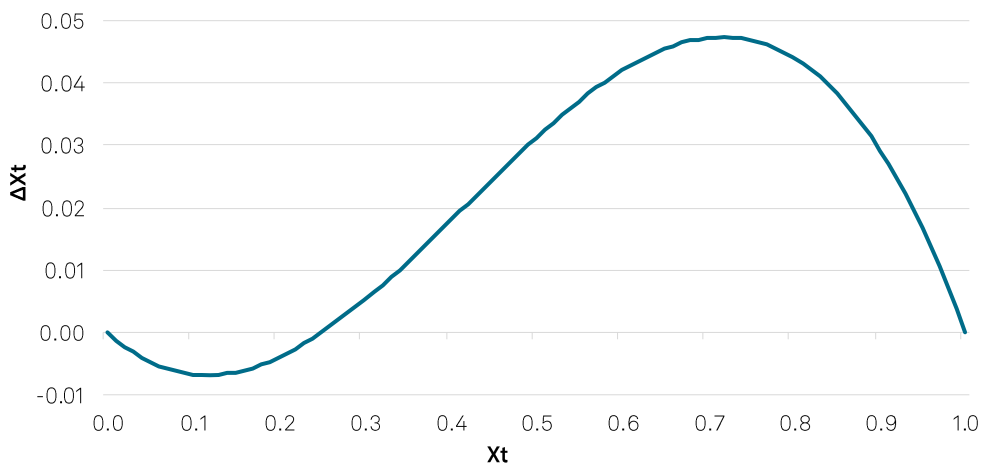
A tipping point is defined as a critical threshold in a dynamic system. Once a system's tipping point is crossed, the properties of that system fundamentally change: It goes from stable (potentially returning to some desired state) to unstable (moving inexorably toward an undesirable state).

### Why Tipping Points Matter

Consider, as an example, a depleted biosphere, such as a rainforest, which can recover if corrective action is taken early. The rainforest has a capacity to repair itself. But if the damage goes on for too long and is too severe--that is, if the system is beyond its tipping point--then it may never be able to recover, and the biosphere will be destroyed.

Chart 1

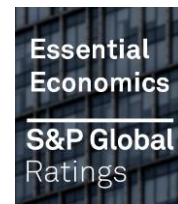
A regeneration function with a tipping point



In this chart, zero represents the totally degraded state, and one represents the pristine state.

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## The Geometry Of Tipping Points

The geometry of tipping points is an example of a relatively simple model generating relatively complex dynamics. Chart 1 plots a cubic function that generates a tipping point.

The horizontal axis shows the value of the state variable at the beginning of time  $t$ , represented as  $X_t$ . Going back to our previous example, think of this as a biosphere. To make the math easier, we define the variable on the interval between zero and one, where zero is the totally degraded state and one is the pristine state. There is a continuum of outcomes between zero and one.

The vertical axis is the change of the state variable during time  $t$ , represented as  $\Delta X_t$ . Critically,  $\Delta X_t$  (the change in the state variable) can be positive or negative, depending on the value of  $X_t$  (time). Specifically, if  $X_t$  is above the tipping point, then  $\Delta X_t$  is positive and the state variable increases over time back toward one. But if  $X_t$  is below the tipping point, then  $\Delta X_t$  is negative and the state variable falls toward zero.

The formula for the function in chart 1 is  $\Delta X_t = rX_t(1-X_t)(L-X_t)$ . This function has only two parameters ( $r$  and  $L$ ), but  $X_t$  appears three times, making it a cubic function (and generating its two-hump shape). The parameter  $r$  represents the height of the curve: A lower  $r$  means a flatter curve and slower adjustment (lower  $\Delta X_t$ ). The parameter  $L$  is the tipping point, which can be anywhere on the horizontal axis, taking a value between zero and one. (In chart 1,  $r$  is 0.5, and  $L$  is 0.25.)

The “action” in this simple model comes from the fact that  $X_t$  is a nonlinear variable: The square and cubic  $X_t$  terms are key to generating the dynamics.

## The Dynamics Of Models With Tipping Points

The system in this example evolves over time according to the following law of motion:  $X_{t+1} = X_t + \Delta X_t$ . In plain English, the value of  $X$  for the next period ( $t+1$ ) is the starting value of  $X$  for this period ( $t$ ) plus the change in its value during the period  $\Delta X$ . For our dynamics,  $X$  can increase over time, decrease over time, or be stable, depending on whether  $\Delta X$  is increasing, decreasing, or zero.

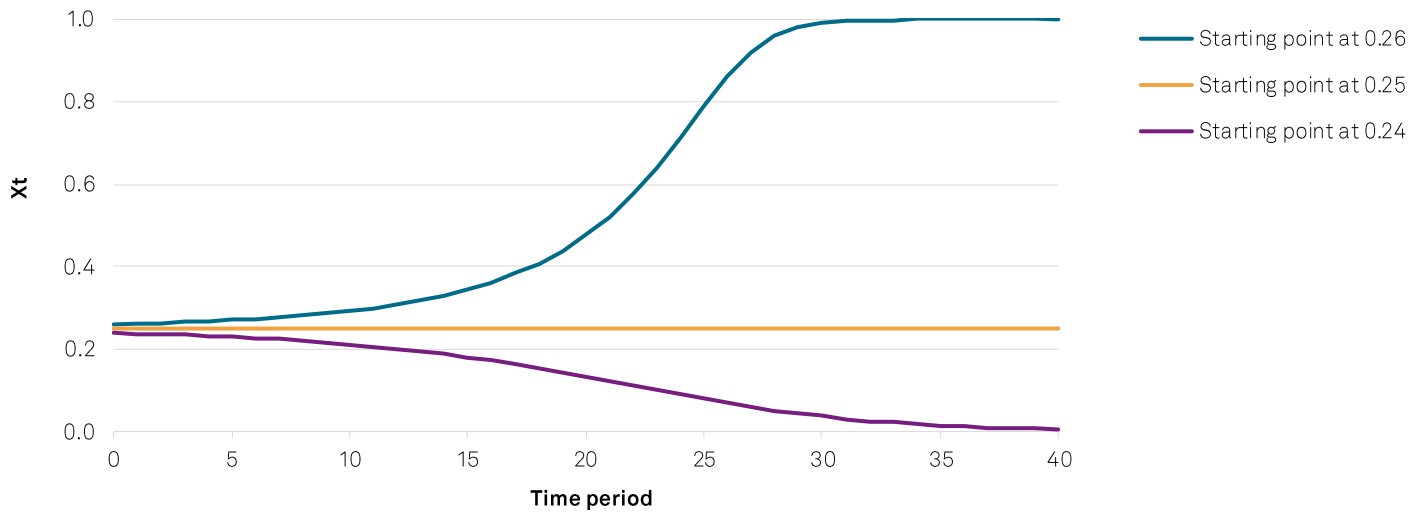
Consider again our biosphere example. The biosphere improves when the state variable is increasing, it deteriorates when the state variable is decreasing, and it's unchanged when the state variable is constant. Whether any particular level of  $X_t$  is “good” is a separate matter.  $X_t = 0.25$  may be an equilibrium, but it could correspond to a state of the biosphere that is unacceptably degraded.

This simple system has three equilibria--three values of the state variable where  $\Delta X_t = 0$ . If the system is at an equilibrium, then it will stay there.

- The equilibria in this model are at one, zero, and the tipping point (0.25). Two of these equilibria are stable while one of them (the one at the tipping point) is unstable.
- If the state variable moves below one or above zero, the system returns to that equilibrium. However, the equilibrium at the tipping point is unstable.
- If the system starts at 0.25, then small changes either way lead to very different outcomes (see chart 2). To see this divergence, [click here](#).

Chart 2

The sensitivity of paths starting near a tipping point



Source: Author's calculations.

This outcome points to the role of tipping points as critical thresholds: Once they're crossed, the dynamic nature of the system changes. Above 0.25, the system returns to the good equilibrium of one (absent any further shocks). But below 0.25, the system moves toward the bad equilibrium of zero. In our biosphere example, recovery is no longer possible once the tipping point is crossed from above.

### The Next Step: Enter The Economy

We've been talking specifically about the environment and its ability to regenerate following shocks, nothing else. It's important to remember that the dynamic explored in this blog post is part of a larger system. We need to add the economy!

Once an economy is introduced and is allowed to interact with the environment, things change. We now have forces pushing in both directions. The environment can regenerate, as shown above, pushing our state variable higher. But economic activity (via emissions and direct impacts on the biosphere) pushes in the opposite direction, forcing the state variable lower.

How these forces interact, and what an equilibrium looks like, will be covered in our soon-to-be released follow-up research. There we will reintroduce a version of our [green growth](#) economy and examine macro-environmental sustainability. This will take into account the regeneration described above and the environmental efficiency of economic output.

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